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The Impact of Central Bank Stock Purchases: Evidence from Discontinuities in Policy Rules*

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Abstract

We trace the impact of central bank stock purchases by exploiting the discontinuity in Bank of Japan's policy rule, which triggers purchases when the stock market index falls below a certain threshold. In normal times, a purchase of 0.01% of market capitalization (a typical size of each intervention) persistently increases the long-term interest rate by around 1.5 b.p. while leaving virtually no detectable impact on stock prices. After the introduction of the yield curve control, which pegs the long-term interest rate to 0%, interest rates stopped responding and stock prices rise by around 0.2% in response to the stock purchases. These results support a theory where *both* stock and bond markets are substantially inelastic.

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1 Introduction

As a new form of "quantitative easing," the Bank of Japan (henceforth, the BoJ) started to purchase stocks in 2010, becoming the largest Japanese stock owner in the world in 2021. Figure 1 shows the cumulative amount of the BoJ's stock holdings as a fraction of Japan's market capitalization. The BoJ continuously kept purchasing stocks, and now holds 6% of market capitalization. No other central banks in the world have been purchasing stocks on such a regular basis. The BoJ explains that the primary goal of this extreme form of quantitative easing is to "reduce the risk premium".

What is the impact of the central bank's stock purchases? Answering this question is important for the following two reasons. First, the BoJ has been always at the frontier of implementing unconventional monetary policy. Policies such as forward guidance or government bond purchases were first implemented by the BoJ in the early 2000s, and a decade later, central banks in the U.S. and Europe have been pursing the same path that Japan has followed. Therefore, learning from the most pioneer of unconventional policies will likely benefit policy makers worldwide in the near future. Second, it provides an ideal laboratory to test new theories of stock market fluctuations. Recently, Gabaix and Koijen (2021) argue that shocks to flows from bonds to stocks can be a dominant source of stock price fluctuations. Assessing the causal impact of central bank's stock purchases simultaneously provides an empirical test of this theory.

We trace the impact of the BoJ's stock purchases by exploiting the discontinuity in the BoJ's policy rule, which triggers purchases when the stock market index falls below a certain threshold. While the BoJ has never publicly stated their practice, it is widely known that the BoJ tends to purchase stock market index precisely on the day when the index falls below a threshold in the morning session. This unique feature enables us to overcome the endogeneity of policy interventions with the regression discontinuity estimator. By comparing days where a stock market index falls slightly below the threshold and slightly above, the policy intervention can be viewed as orthogonal to the underlying economic fundamentals.

In the entire sample, we first show that the policy had a large impact on *both* the stock prices and the long-term government bond interest rates. In response to an average size of intervention, which amounts to 0.01% of stock market capitalization, our estimates indicate that the stock prices rise by around 0.3-0.4% on the same day, and by around 0.1% on the following day of the intervention. Perhaps surprisingly, the 10-year Japanese government bond (JGB) yield also increases by around 0.5 basis point (b.p.) on the same day and the following day. These results seem to be persistent, lasting at least several days.

We then argue that the above results mask stark underlying heterogeneity that depends on the presence of yield curve control (YCC) by the the BoJ. In the middle of 2016, the BoJ introduced another form of unconventional monetary policy, the so-called yield curve control,

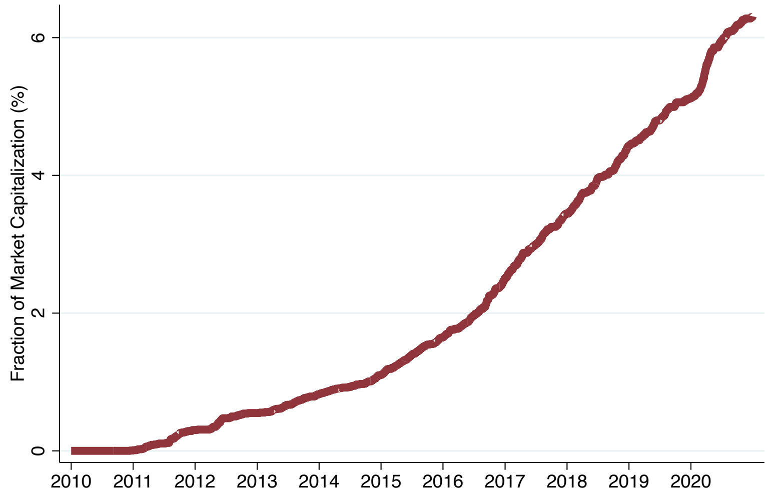

Figure 1: Cumulative ETF Purchases by the BoJ

Notes: Figure 1 plots the cumulative amount of ETF purchases by the BoJ from 2010 to 2020 as a fraction of market capitalization.

which pegs the 10-year JGB yield to around 0%. Since then, long-term interest rates have stabilized at around 0%. Given this, we expect that the response of long-term interest rates to be substantially different before and after the introduction of YCC. We therefore split the sample and re-examine the effects.

We find that before the introduction of YCC, in response to the BoJ's stock market purchases, the long-term interest rates strongly and persistently rise after the intervention, while leaving virtually no impact on the stock price the next day. The long-term interest rates rise by 1.5 b.p. on the same day, which persists at least several days. In contrast, while stock prices rise by around 0.3-0.4%% on the same day, they revert back to zero the morning of the next day and remain there. We are unable to reject the null that the BoJ's stock purchases left no impact on stock prices in the following days, although the standard errors are wide.

After the introduction of YCC, the long-term interest rate stopped responding, and instead, stock prices rose persistently in response to the Bank of Japan's intervention. The effect on the long-term interest rate is precisely estimated zero. The stock price responds by around 0.3-0.4%% on the same day, and this persists for at least several days after the intervention.

We then turn to a theoretical framework to interpret our empirical findings and argue that our empirical results support a model in which *both* stock and bond markets are substantially inelastic. Our empirical results reject a frictionless model, which predicts the neutrality with respect to central bank stock purchases (Wallace, 1981). Our results are also inconsistent with the model presented in Gabaix and Koijen (2021), in which the stock market is inelastic but the bond market is perfectly elastic. In their model, the stock market is inelastic because stock traders face various frictions in flexibly adjusting their portfolio, such as institutional mandates or inattention. This predicts the large increase in stock prices and zero effect on interest rates in response to the sudden inflow into the stock market. However, our empirical results do not support this model. Instead, we draw on our findings to propose a model in which both stock and bond markets are inelastic. Central bank stock purchases are a swap between stocks and bonds in the market. When the bond market is inelastic, the outflow of bonds reduces the price of bonds, resulting in an increase in interest rates. While inflow into stocks tends to increase the stock price, the increase in interest rates acts as a countervailing force. Our empirical results from the period before YCC implies that these two forces offset each other, leaving no effect on stock prices. We then show that once the interest rate becomes fixed, through YCC, only the former force remains, which leads to an increase in stock prices.

Our theoretical framework demonstrates that these forces are summarized by three intuitive sufficient statistics: (i) stock market inelasticity; (ii) bonds market inelasticity; and (iii) the interest rate sensitivity of stock prices. We show that these sufficient statistics can be identified via our reduced-form estimates, providing a structural interpretation of our reduced-form analysis. This should serve useful for future researchers disciplining or calibrating their theoretical models.

Through the lens of the model, our estimates of stock market inelasticity, which we define as the impact on stock prices from an inflow into the stock market *holding interest rate fixed*, is several times higher than the estimates provided by Gabaix and Koijen (2021). This implies the frictions in the stock market is large. However, our estimates indicate the frictions in the bond market is even larger, in the sense that, without an explicit constraint on the interest rate adjustment, flows between stocks and bonds will mostly end up moving bond prices rather than stock prices. A broader message of our paper is that is crucial to take into account frictions in both the bond and the stock market at the same time, while the literature has been typically considered one friction at a time.[1]

1.1 Literature

To the best of our knowledge, we are the first to uncover the aggregate causal impact of central bank stock purchases. Since the BoJ purchased a certain stock market index rather than the another, many studies exploit difference in weights in the BoJ purchase basket to identify the relative price impacts (Barbon and Gianinazzi, 2019; Charoenwong et al., 2021; Harada and Okimoto, 2021; Adachi et al., 2021; Katagiri et al., 2022). In contrast, our empirical strategy allows us to focus on the aggregate effect. Various studies (Shirota, 2018; Fukuda and Tanaka, 2022; Chung, 2020) assume selection on observables and use the unexplained policy variation that remains after conditioning on observables for identification. However, any endogeneity

[1]See Krishnamurthy and Vissing-Jorgensen (2012) for bond market frictions and Gabaix and Koijen (2021) for stock market frictions.

due to unobservables will bias the estimates. Our identification assumptions are substantially weaker than any of these studies.

More broadly, we contribute to the large literature studying the effect of the central bank asset purchases, so called "quantitative easing" (e.g., Krishnamurthy and Vissing-Jorgensen, 2011; Chodorow-Reich, 2014; Gorodnichenko and Ray, 2017). These studies have focused on central bank purchases of long-term government bonds or mortgage backed securities, which are the swaps between one type of bonds (e.g., long-term bonds) with another (e.g., reserves). Our focus is conceptually distinct from them because central bank stock purchases are swaps between stocks and bonds in the economy.

Through the lens of our theoretical model, we argue that our empirical setup provides joint estimates of two important structural parameters: stock market inelasticity and bond market inelasticity. While growing studies (Gabaix and Koijen, 2021; Da et al., 2018; Hartzmark and Solomon, 2021; Li et al., 2021) estimate how the flows from bonds into stocks impact stock prices, we argue it is important to jointly take into account how bond prices are impacted by such flows. Our empirical results suggest that such flows increase the interest rates, which counteract the upward pressure on stock prices. Our empirical estimates of stock market inelasticity when interest rates can respond is indistinguishable from zero. However, the estimates of stock market inelasticity when the interest rate is fixed is several times higher than existing estimates. In this respect, we synthesize two strands of literature: one on bond market inelasticity (e.g., Krishnamurthy and Vissing-Jorgensen, 2012) and the other on stock market inelasticity (Gabaix and Koijen, 2021). We argue it is critical to take into account these two views jointly.

While there are many studies to isolate quasi-experimental variation in monetary policy (Romer and Romer, 2004; Cochrane and Piazzesi, 2002; Angrist et al., 2018; Nakamura and Steinsson, 2018), our approach is unique in utilizing the discontinuity in policy rule. The closest to our approach is the one in Kuersteiner, Phillips, and Villamizar-Villegas (2018), who also utilize a discontinuous policy rule to investigate the effectiveness of sterilized foreign exchange interventions in Columbia. Our approach differs not only in terms of the empirical contexts but also in methodology, as we employ the technique of regression discontinuity with unknown discontinuity points (Porter and Yu, 2015).

2 Data

Our primary goal is to measure the impact of central bank stock purchases on financial markets. We focus on the time period starting in October 2010 (the start of the BoJ stock market intervention) toward the end of 2020. We obtain dates and amount of stock purchases for each the BoJ's interventions from the BoJ website at https://www3.boj.or.jp/market/jp/menu_etf.htm.

Figure 1 shows the amount of ETF purchases over time. The BoJ started the stock market purchases in December 2010, and the frequency and the amount of purchases grew over time. By the end of 2020, the BoJ held over 6% of the stock market capitalization in Japan. The amount of ETF purchases is normalized by the stock market capitalization], which we obtained from Japan Exchange Group Data Cloud.

The BoJ publishes the amount of ETF purchases the morning following the intervention. Based on the trading volume, it is widely considered that the BoJ submits an order during lunchtime, although the BoJ has never made this practice public.[2] Therefore, investors potentially face uncertainty about whether the large inflow into the stock market reflects the BoJ's intervention or other factors within the day of the intervention. For this reason, we prefer to take our empirical estimates of the one-day horizon as our benchmark estimates.

To measure the response of the stock market, we use tick-by-tick data on the Tokyo Stock Price Index (henceforth, TOPIX), which is the index of the Tokyo Stock Exchange in Japan, tracking all domestic companies of the exchange's first section. We obtain these data from Japan Exchange Group Data Cloud. To measure the response of the long-term interest rate, we use the tick-by-tick Japanese Government Bond yield data, which we obtained from Refinitiv Japan. Since some observations are missing in the Refinitiv data, we supplement those missing observations with the tick-by-tick data from Bloomberg.[3]

3 Research Design

Our primary goal is to identify the effect of the BoJ's stock purchases on financial markets. We consider the following econometric model

$$\Delta y_{t+l,h} = \beta_{l,h} \times ETF_t + \Gamma'_{l,h} \mathbf{X}_t + \epsilon_{t+l,h}, \tag{1}$$

where $\Delta y_{t+l,h} \equiv y_{t+l,h} - y_{t,0}$ is the change in variable y (e.g. the log of stock prices) from the end price of the morning ($h = 0$) session on day t to time h on l days later, ETF_t is the amount of stock market purchases by the BoJ relative to the stock market capitalization of Japan, $\mathbf{X}_t$ is the vector of controls, and $\epsilon_{t+l,h}$ contains the unmodeled determinants of the outcome variable. We are interested in estimating $\beta_{l,h}$, which measures the impact of central bank stock purchases at time h at l days after day t . We choose this simple linear model for expositional purposes. In Appendix A.1, we consider a non-linear model of (1) and present more technical interpretation of the estimated parameter as in Angrist and Imbens (1995).

An obvious concern for estimating equation (1) via OLS is reverse causality. We expect that

[2] See Harada and Okimoto (2021), for example.

[3] The results are very similar when we only use the data either from Refinitiv or from Bloomberg. The two datasets, when they overlap, are highly correlated with each other, with R^2 exceeding 99.98%.

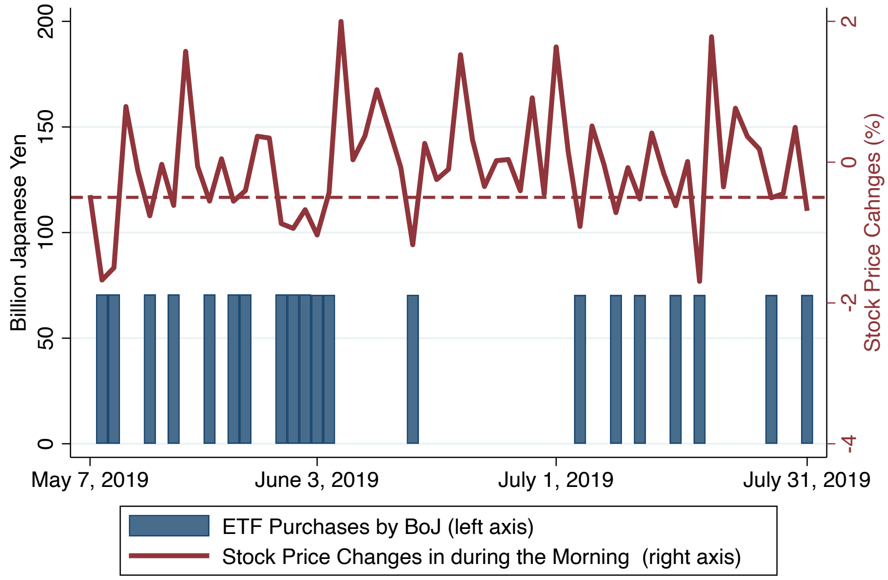

Figure 2: An Example of Cutoff Policy Rule

Notes: Figure 2 illustrates the cut-off policy rule by showing the percentage TOPIX changes and the BoJ (Bank of Japan) purchase amount for each day from May 2019 to July 2019. The solid red line shows the TOPIX changes in the morning session, and the dashed red line is the estimated cutoff of 0.25%. The bar shows the amount of purchases for each intervention in billions of Japanese Yen (approximately 10 million US dollars).

the central bank is more likely to intervene when the stock market performs poorly. This leads to the downward bias of the OLS estimates of $\beta_{l,h}$.

To solve this endogeneity problem, we propose a regression discontinuity based identification strategy that builds on the observation that the BoJ intervention appeared to follow a cut-off rule. It has been commonly argued among the media that the BoJ appeared to intervene on the day when the value of TOPIX falls below a certain threshold in the morning. For example, Financial Times write "the central bank has tended to step in whenever the TOPIX index has lost more than 0.5 percent in the morning session".[4] In fact, Figure 2 shows that from May to July 2019, the BoJ indeed followed a strict rule to intervene when the stock market index falls more than 0.5% in the morning session. The BoJ intervenes when the index falls slightly below the 0.5% threshold, while it does not intervene when the index falls slightly above the threshold.

Suppose for the moment that such a cut-off is known. Then, we can apply a standard regression discontinuity design. Formally, we assume the policy rule takes the following form,

[4] Financial Times, "Bank of Japan backs away from ETF buying scheme," (March 23, 2021), https://www.ft.com/content/a654d1c9-7126-4587-8de6-ed15f567455f.

in which the amount of ETF purchase, ETF_t, is given by

$$ETF_t = ETF_{-,t}(\Delta p_t)\mathbb{I}(\Delta p_t < c_t) + ETF_{+,t}(\Delta p_t)\mathbb{I}(\Delta p_t \geq c_t), \tag{2}$$

where Δp_t is the log-changes in the TOPIX value in the morning, c_t is the cut-off, $ETF_{-,t}$ and $ETF_{+,t}$ are some random functions of the ETF puchase at day t that represent different policy rules depending on whether Δp_t is above or below the cutoff. We assume (i) $\mathbb{E}[\epsilon_{t+l,h}|\Delta p_t, \mathbf{X_t}]$ is continuous at $\Delta p_t = c_t$, (ii) $\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]$ and $\lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]$ exist, and (iii) $\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}] \neq \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]$. Under these assumptions, it follows that

$$\frac{\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p, \mathbf{X_t}] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p, \mathbf{X_t}]}{\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]} = \beta_{l,h}. \tag{3}$$

As recommended by Hahn, Todd, and Van der Klaauw (2001) and Porter (2003), we can devise local linear regression estimators for the left-hand side to obtain an estimate of $\beta_{l,h}$. Imbens and Lemieux (2008) pointed out that this is numerically equivalent to a two-stage least squares estimator with properly defined instruments and weights. The advantage of their formulation in our context is that it is easy to accommodate heteroskedasticity and auto-correlation. We use the optimal bandwidth proposed by Calonico, Cattaneo, and Titiunik (2014) and estimate $\beta_{l,h}$ using two-stage least squares and report Newey-West standard errors.

The difficulty in implementing the above approach is that the cut-off is not necessarily known. While it was apparently known by the public that the BoJ followed a particular cut-off rule in some periods, it is sometimes not in other periods. In order to formally investigate the hypothesis, we aim to estimate the cut-off with the presumption that the BoJ follows a cut-off rule, following the approach proposed by Porter and Yu (2015). Porter and Yu (2015) propose a method to estimate the discontinuity point and show that there is no loss in efficiency with the regression discontinuity estimator using the estimated cutoff. In implementing this approach, we proceed as follows. We first split the sample period to allow time-variation in the policy rule. We assume the cut-off is a constant within the sample split. Then, in each of the sample splits, we consider a set of possible cutoffs, $\mathbb{C} \equiv \{\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_K\}$. For each $\bar{c} \in \mathbb{C}$, we estimate the jump of $\Pr_t(ETF_t > 0|\Delta p)$ around $\bar{c}$, which is

$$J_t(\bar{c}) \equiv \lim_{\Delta p \uparrow \bar{c}} \Pr_t(ETF_t > 0|\Delta p) - \lim_{\Delta p \downarrow \bar{c}} \Pr_t(ETF_t > 0|\Delta p).$$

We select $\bar{c}$ that maximizes square of the jump, $J_t^2(\bar{c})$: $c_t^* \in \arg\max_{\bar{c} \in \mathbb{C}} J_t^2(\bar{c})$.

We implement the above approach with the following specifications. First, we consider the split of the sample period based on the BoJ's announcement regarding the ETF purchases. The BoJ made six announcements that state the changes in the target amount of ETF purchases on

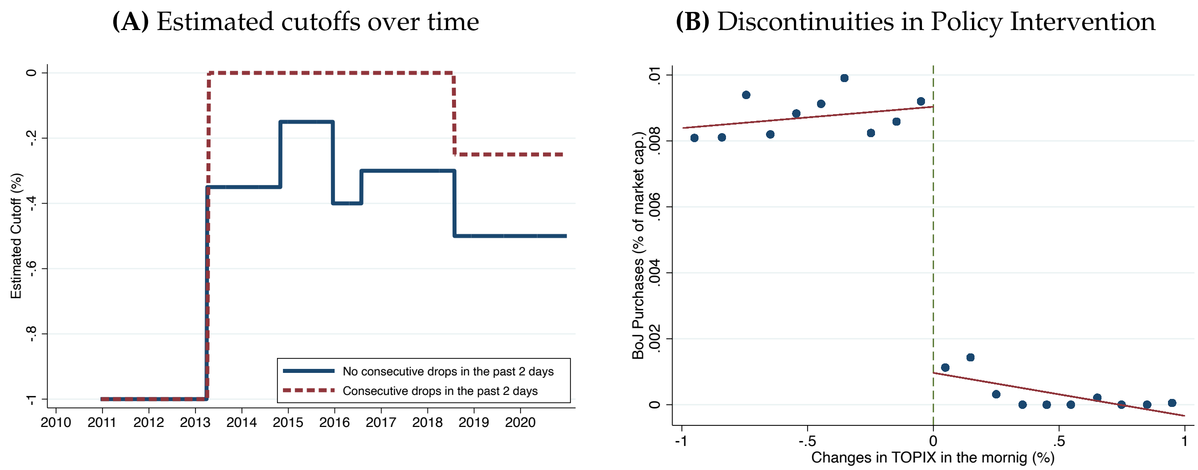

Figure 3: Estimated Cutoffs and Discontinuities around the Cutoffs

Notes: Figure 3A plots the path of estimated cutoffs over our sample period. Figure 3B shows the discontinuity in the amount of the BoJ stock purchases in the range of -1% to 1% around the estimated cutoff. Each dot represents the binned scatter plot with 0.1% bin-width, and the red line represents the linear fit on each side of the cutoff.

March 4, 2013, October 31, 2014, December 18, 2015, July 29, 2016, July 31, 2018, and March 16, 2020. We further split each period between the two announcements based on whether the TOPIX closing price falls relative to the opening price for the past two consecutive days. We make this choice based on widely held claims in the media,[5] and we indeed found this has a strong explanatory power. Second, we consider the set of potential cutoffs ranging from -1% to 0% with 0.05% intervals. We estimate the jump of $\Pr_t(ETF_t > 0|\Delta p)$ around the potential cutoffs using the local linear regressions with the optimal bandwidth computed from Calonico, Cattaneo, and Titiunik (2014).

Relevance of cutoffs. Figure 3A shows the path of estimated cutoffs. The estimated cutoffs align well with the widely held consensus. During 2010-2013, it is widely believed that the BoJ followed a so- called “1% rule”, in which the BoJ buys ETFs whenever the TOPIX falls more than 1% in the morning session,[6] and our estimates confirm this view. Starting in April 2013, the BoJ appears to use the different cutoffs depending on whether the daily changes in TOPIX are negative in the past two consecutive days. Since March 2018, the cutoffs appear to be 0.5% when there is no consecutive fall in the past two days, which is again consistent with the so-called “0.5% rule.”

[5]See, for example, Bloomberg article “The BoJ’s ETF Purchase Conditions Likely to Ease if Stocks Continue to Fall” (written in Japanese) (`https://www.bloomberg.co.jp/news/articles/2020-07-22/-0-3-kcwteezj`).

[6]For example, Nikkei Asia writes “the BoJ was widely thought to be following an unwritten rule, dubbed the 1% rule: it would buy ETFs when the Topix index of all issues on the first section of the Tokyo Stock Exchange fell more than 1% in the morning session.” (https://asia.nikkei.com/Business/Finance/BOJ-steps-up-REIT-buying-scales-back-ETF-purchases)

Figure 3B shows the binned scatter plot of the size of the BoJ intervention against the changes in TOPIX in the morning session on the same day relative to cutoffs. We confirm that there is a discrete jump in the size of the BoJ interventions around zero. The implied jump in the overall sample is 0.83% of the market capitalization with the standard error of 0.0005%. The Cragg-Donald F statistic is 1821, and the Kleibergen-Paap F statistic is 261, which sweeps out weak identification concerns. This discontinuity comes from the discontinuity in the likelihood of an intervention with a jump in probability of intervention of 86% with a standard error of 0.02%. Importantly, we find strong evidence of discontinuity in any single split of the sample.[7]

Manipulation test. A natural concern for discontinuity-based research design is the manipulation around the cutoff. While it is unlikely that investors are able to manipulate the stock price, we formally test the presence of manipulation using the methodology proposed by Cattaneo et al. (2020) in Appendix A.3, and the density plot is shown in Figure B.2. We do not find evidence of manipulation.

(Dis)continuity in ETF purchases across days. $y_{t+l,h}$ is clearly affected by the BoJ's ETF purchases up to l days later. Therefore, if falling below the cutoff today is correlated with the future and past purchases, our empirical estimates cannot be interpreted as the causal effect of the BoJ's one-time ETF purchases. In order to address this concern, in Appendix A.4 we test the discontinuity in the amount of ETF purchases around the cutoff across days. Figure B.3 shows the estimates of discontinuity of the amount of ETF purchases at date $t + l$ around the cutoff at day t. Reassuringly, we find significant discontinuity only at $l = 0$. Therefore, our identified effects are the causal effects of the BoJ's one-time ETF purchases and are not contaminated by future or past ETF purchases.

4 Empirical Results

Armed with the estimates of cutoffs, we implement the regression discontinuity design to assess the impact of the BoJ ETF purchases on the financial market. We report the following three main results: (i) Bank of Japan's stock purchases increase both stock prices and long-term interest rates in the overall sample period, (ii) in periods before Bank of Japan introduces yield curve control, there is no evidence of stock price increase, but robustly increases the long-term interest rate, and (iii) after the introduction of yield curve control, long-term interest rate stopped responding, and stock market robustly increases.

[7] We report the discontinuity for each sample split in Appendix A.2. .

(A) Discontinuity in Stock Return within a Day **(B)** Discontinuity in 10 Year JGB Yield within a Day

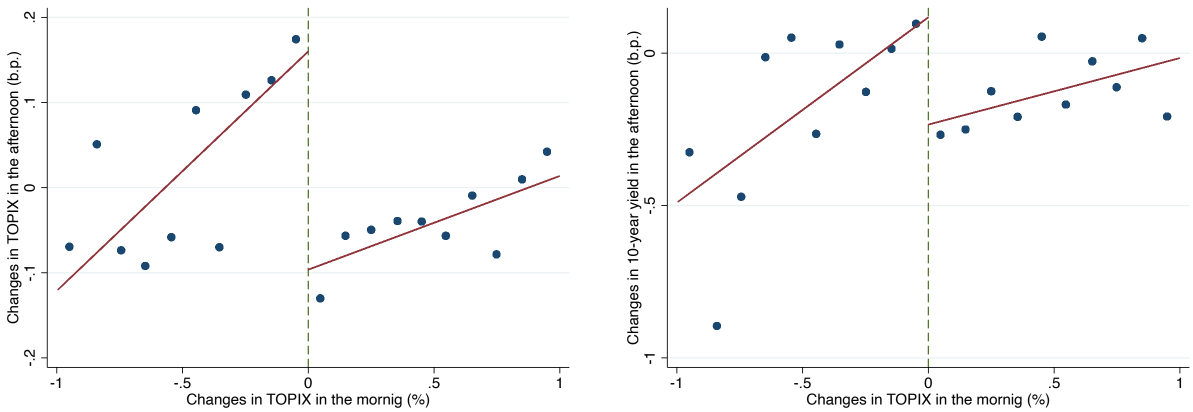

Figure 4: Discontinuities in Stock Returns and Long-Term Interest Rates

Notes: Figure 4A shows the binned scatter plot of the log-changes in TOPIX in the afternoon (from 11PM to 3PM) against the changes in TOPIX in the morning session relative to the cutoff. The bin width is 0.1%. The line represents the best fit from the linear regression, with the shaded area being a 95% confidence interval. Figure 4B is analogous to Figure 4A, with the vertical axis being the changes in the 10-year JGB Yield in basis point (b.p.) in the afternoon (from 11PM to 3PM).

4.1 Homogenous Effect for the Entire Sample Periods

Figure 4A first assesses whether a discontinuity in policy intervention leads to a discontinuity in stock prices changes. It reports the binned scatter plot changes in TOPIX in the afternoon (from 11AM to 3PM) against the changes in TOPIX in the morning relative to the estimated cutoff. The figure shows that the stock prices were around 0.2% higher when the TOPIX falls slightly below the cutoff in the morning than when it falls slightly above the cutoff. Since the BoJ submits the order to purchase ETF during the lunch break, this suggests that the BoJ intervention had a large impact on the stock prices within the day. The magnitude is large, given that the BoJ purchased around 0.01% of market capitalization in each of the interventions on average.

Figure 4B focuses on the 10-year Japanese government bond yields as an outcome variable. Perhaps surprisingly, we see discontinuity also in the long-term interest rate. The long-term interest rate is 4 basis points higher on the left side of the cutoff than on the right side. Later, we argue that through the lens of the theoretical model, this evidence supports the notion that the bond market is inelastic. Intuitively speaking, central banks swap the bonds with stocks. As there are more supplies of bonds in the economy, the bond prices fall if the investors' demand for bonds are downward sloping. The results so far concern the price changes within a day, and therefore it could be the case that the stock and bond prices revert back to the original level on the following day. We next systematically asses the price impacts for various horizons.

Figure 5A and 5B plot the impulse response functions of stock prices and bond prices. Formally, we plot we estimate β_h in equation (1) for each h, where $h = 0$ is 11AM of the day of the

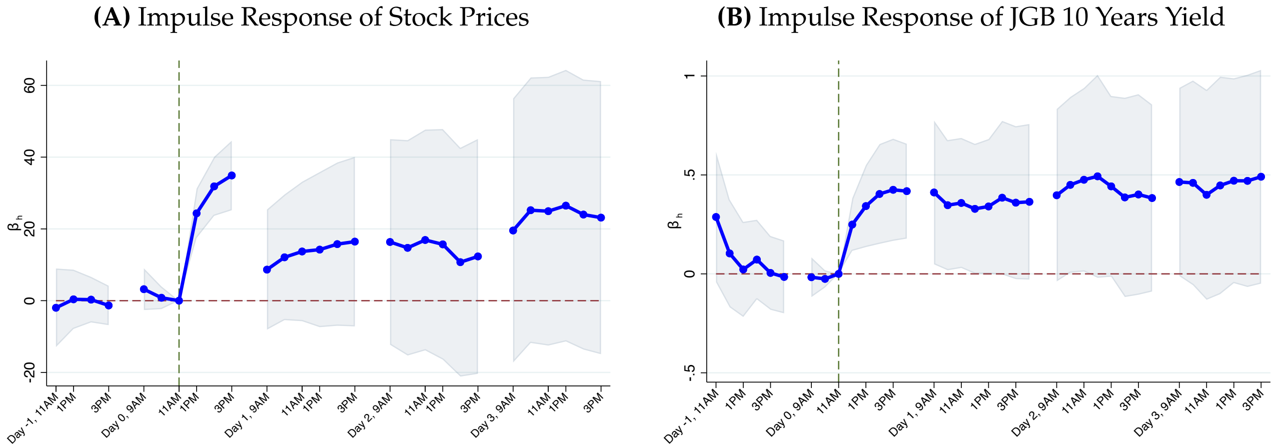

Figure 5: The Impact on Stock Prices and Long-Term Interest Rates

Notes: Figure 5A shows the impulse response function of stock prices by plotting coefficient β_h in equation 1. The coefficient measures the log-changes in stock prices in response to stock purchases of 1% of market capitalization. Figure 5B is analogous to Figure 5A and shows the impulse response of the 10-year JGB yield. The coefficient measures the percentage point changes in the yield in response to stock purchases of 1% of market capitalization. In all figures, the shaded areas represent 90% confidence intervals, which account for heteroskedasticity and autocorrelation.

intervention with one-hour interval of market hours. In Figure 5A, we see an immediate and large stock price response in the afternoon of the intervention. It implies that 1 stock purchases of 0.01% of market capitalization, a typical size of the intervention, increases the stock value by 0.4%. This coefficient is highly statistically significant. Over the next five days, the coefficient is about halved and the standard error is wider, but it does not revert back to zero. Reassuringly, we do not find evidence of pre-trends, which is consistent with the continuity assumption on the error term.

Figure 5B shows that the 10-year JGB yield also sharply rises following the intervention. The effect appears to be quite persistent, and it remains statistically significant even five days after the intervention. The magnitude is again sizable. In response to a typical size of the purchases (0.01% of market capitalization), the 10-year JGB yield rises by around 0.4-0.5 basis point.

We have shown that the central bank stock purchases have quantitatively large impacts on both the stock and bond markets. In what follows, we argue that these average effects mask an important underlying heterogeneity.

4.2 Heterogenous Effect and Yield Curve Control

Figure 5B showed that the stock market purchases are accompanied by the rise in the long-term interest rate. In standard theoretical models, the rise in interest rate leads to a drop in stock prices. We therefore expect that the ability of the interest rate to respond is critical in

Figure 6: 10 Year JGB Yield

Notes: Figure 6 shows the path of 10-year JGB (Japanese Government Bond) yield over time, where the red vertical dashed line (September 21, 2016) denotes the start of the yield curve control.

determining the stock price responses to the central bank stock purchases.

The BoJ's other unconventional policy, the so-called "yield curve control," provides an ideal laboratory to explore this hypothesis. On September 21, 2016, the BoJ introduced an explicit target for the 10-year Japanese government bond yield at 0%. Figure 6 indeed shows that the long-term rate stabilized at around 0% since the introduction of the yield curve control. The daily standard deviation of the long-term rate is 0.37% before the introduction of yield curve control, but it falls to 0.08% after the introduction. If the BoJ does its best to stabilize the long-term interest rate at 0%, we would expect to see a much smaller response of the long-term rate in response to the stock purchases. To test this, we split our sample periods before and after the introduction of yield curve control and re-do our analysis.

Figures 7A- 8B show the main results of this paper. Figures 7A and 7B plot the impulse response of stock prices before and after the introduction of yield curve control, repsectively. We find no evidence that the stock market responds positively after the BoJ intervention before the introduction of yield curve control after one day, although the standard error is large. In a stark contrast, stock prices persistently rise economically and in a statistically significant manner under the yield curve control. Quantitatively, a 1% stock purchase by the BoJ causes around a 20-30% increase in stock prices within at least several days after the intervention.

Figures 8A and 8B explain why. The long-term interest rate responds positively before the yield curve control. Quantitatively, a 1% stock purchases by the BoJ causes around a 1-2% increase in the long-term rates, and the effect is statistically significant. However, under the yield curve control, long-term rates stopped responding and the effect is precisely estimated zero.

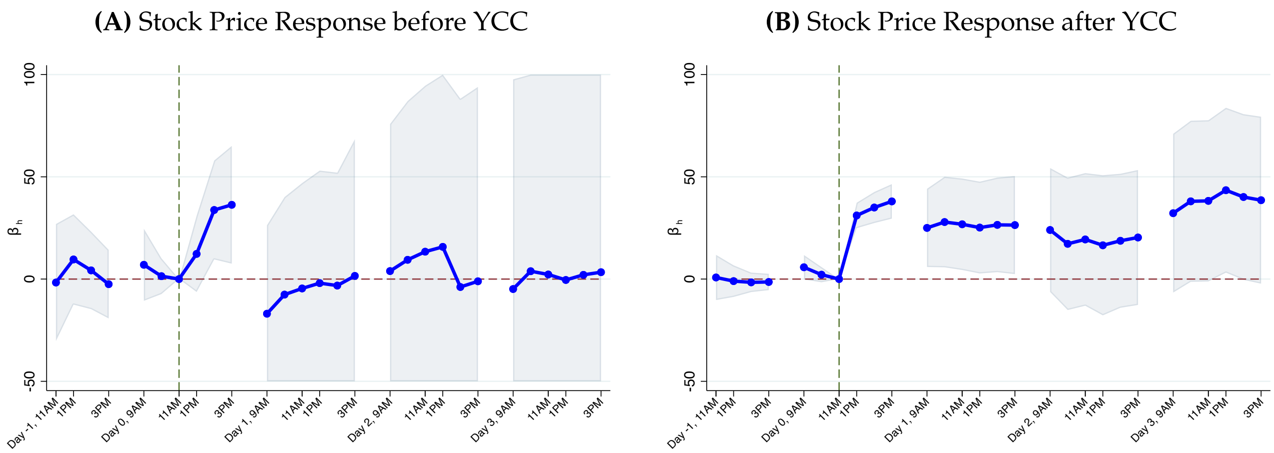

Figure 7: Heterogenous Stock Price Responses before and after Yield Curve Control

Notes: Figures 7A and 7B show the impulse response of stock prices separately estimated before and after the yield curve control, which are analogous to Figure 5A.

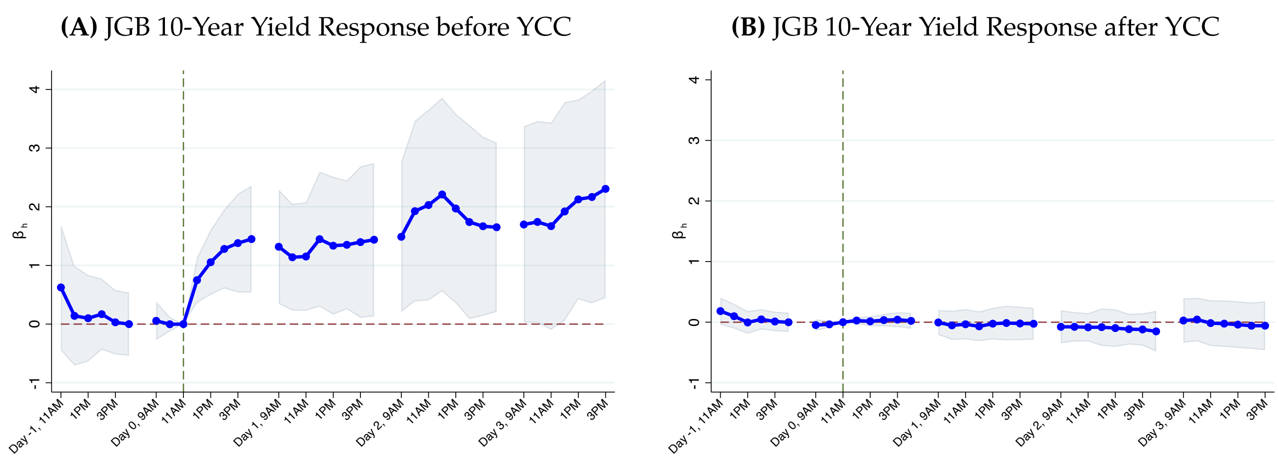

Figure 8: Heterogenous Interest Rate Responses before and after Yield Curve Control

Notes: Figures 8A and 8B show the impulse response of the 10-year JGB yield separately estimated before and after the yield curve control, which are analogous to Figure 5B. In all figures, the shaded areas represent the 90% confidence interval, which accounts for heteroskedasticity and autocorrelation.

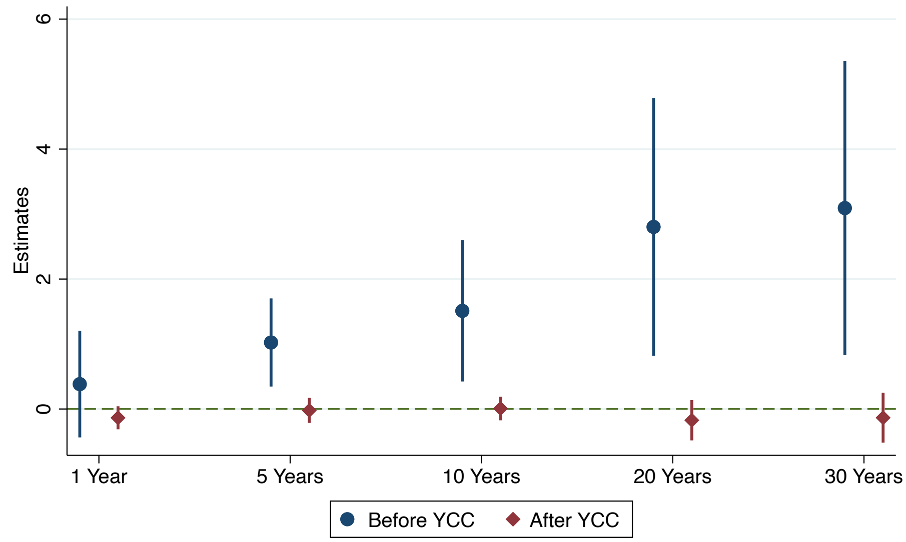

Figure 9: Response of Other Maturities

Notes: Figure 9 shows the response of the JGB yield across different maturities from 11AM of the day of the intervention to 9AM of the next day. The circle dot represents the point estimates before the yield curve control, and the diamond dot represents the point estimates after the yield curve control. The coefficient measures the percentage point changes in the JGB Yield in response to the purchase of 1% of market capitalization. The vertical lines represent the 90% confidence interval, which accounts for heteroskedasticity and autocorrelation.

4.3 Bond Yield Responses Across Different Maturities

We show that the effect on interest rate is not specific to the 10-year JGB yield, but rather is wide spread across different maturities.

Figure 9 shows the point estimates of the effect on JGB yield across maturities of 1, 2, 5, 10, 20, and 30-years. Before the yield curve control, the yield on all maturities rose, but with a larger effect on longer maturities. Our preferred interpretation is that the zero lower bound on policy rate has been binding during this period, and therefore the shorter maturity bonds had less room to respond relative to longer maturity bonds. After the yield curve control, all interest rates entirely stopped responding. Even though the yield curve aimed to specifically control the 10-year yield, it can prevent the response of other maturities because they are interconnected through arbitrage. For example, it is the natural prediction that arises from the preferred habitat model of term structure by Vayanos and Vila (2021).

4.4 Robustness

Table 1 conducts a battery of robustness checks and shows that our results are robust to various alternatives to the baseline specifications. In rows 1 and 2, we show that the results are not sensitive to changing the bandwidth of the regression discontinuity estimator.[8] Row 3 uses the quadratic local polynomial regression instead of the linear. Row 4 controls the amount of the BoJ's purchases in the past two days. This addresses the concern that if the stock price changes are likely to fall on one side of the cut-off over consecutive days, our estimator confounds the effect from the past and the future interventions. Reassuringly, the results are not sensitive to this control. This is not surprising given that there is no evidence that falling below the cutoff today causes the future and past ETF purchases, as we formally show in the Appendix A.4. Row 5 and row 6 control the past stock market return and changes in the long-term interest rate over the past two days. Finally, in row 7, we drop observations one week before and after the dates when the cutoff changed. This addresses the concern that the changes in cutoff could be endogenous to the underlying economic fundamentals or the cutoff change may contain signals about future policy stances of the BoJ. Collectively, our results appear to be virtually unchanged to any of the above modifications.

4.5 Placebo Tests

We conduct placebo tests by testing the presence of discontinuity in outcome variables around an arbitrary cutoff for which we do not expect to find any discontinuity. One might worry that our results are not driven by the BoJ's policy intervention, but rather some other factors such as investors sentiments thatdiscontinuously respond to the stock price changes in the morning session. For example, investors might speculate that the stock market should see a stronger rebound when stock prices fall below 0% in the morning session.

To address this concern, for each value of $c \in \{-1\%, -0.95\%, \ldots, -0.05\%, 0\%\}$, we test whether there is discontinuity in our outcome variables when the stock prices fall below the threshold c. We estimate

$$\Delta y_{t+l,h} = \gamma_{l,h} \times \mathbb{I}(\Delta p_t < c) + e_{t+l,h}, \tag{4}$$

where Δp_t is the percentage change in TOPIX in the morning session. When estimating, we exclude periods for which the cutoff is identical to c under consideration. We are interested in the estimates of $\gamma_{l,h}$, and we expect that $\gamma_{l,h}$ to be indistinguishable from zero for any value of c.

Figures 10A and 10B show the estimated value of $\gamma_{l,h}$, together with the 90% confidence interval. Reassuringly, we find that the estimates of $\gamma_{l,h}$ are indistinguishable from zero in almost all cases. Even if they are significant, the estimates are the opposite sign from our baseline estimates. Moreover, in all cases, the estimates are far enough from our baseline estimates that use the actual cutoff. These results suggest that our results are indeed driven by the policy intervention itself.

[8] Figure B.5 in the Appendix more systematically explore robustness with respect to the choice of bandwidths. We find that the results are virtually unchanged for a wide variation in the bandwidths.

Panel A. Stock Price Response

	All		Before YCC		After YCC	
	Same Day	Next Day	Same Day	Next Day	Same Day	Next Day
0. Baseline	34.90	8.65	36.31	-16.95	35.24	22.23
	(5.89)	(10.18)	(17.50)	(26.27)	(4.68)	(11.13)
1. Narrower	41.65	16.22	30.14	-21.25	47.26	33.08
Bandwidth	(7.16)	(16.09)	(22.18)	(39.57)	(7.15)	(16.25)
2. Wider	29.49	6.07	30.63	-2.44	29.41	15.66
Bandwidth	(4.84)	(9.36)	(13.46)	(20.74)	(4.69)	(9.14)
3. Polynominal	43.40	16.98	41.13	-22.70	47.24	30.63
Order 2	(7.47)	(15.64)	(21.64)	(42.42)	(7.14)	(14.39)
4. Control Past	39.50	12.56	54.57	-16.44	37.32	29.65
Interventions	(8.09)	(15.10)	(25.67)	(34.04)	(6.61)	(14.48)
5. Control Past	34.91	8.62	36.15	-17.33	35.18	20.51
Stock Returns	(5.89)	(10.20)	(17.70)	(26.21)	(4.77)	(11.76)
6. Control Past	35.96	6.63	40.91	-23.41	35.06	21.90
10-Year Yield	(5.84)	(9.94)	(18.33)	(25.42)	(4.67)	(11.13)
7. Drop Around	34.96	8.00	31.21	-20.53	35.07	24.56
the Cutoff Changes	(6.05)	(10.92)	(16.52)	(28.22)	(5.52)	(12.45)

Panel B. JGB 10-Year Yield Response

	All		Before YCC		After YCC	
	Same Day	Next Day	Same Day	Next Day	Same Day	Next Day
0. Baseline	0.47	0.37	1.52	1.41	0.04	-0.00
	(0.18)	(0.23)	(0.59)	(0.68)	(0.07)	(0.13)
1. Half	0.54	0.43	1.96	1.96	-0.00	-0.13
Bandwidth	(0.22)	(0.30)	(0.84)	(0.96)	(0.09)	(0.20)
2. Wider	0.40	0.35	1.28	1.05	0.01	-0.04
Bandwidth	(0.16)	(0.19)	(0.47)	(0.50)	(0.06)	(0.10)
3. Polynominal	0.52	0.43	1.81	1.75	0.07	0.01
Order 2	(0.22)	(0.27)	(0.77)	(0.91)	(0.09)	(0.18)
4. Control Past	0.47	0.37	1.73	1.79	-0.06	-0.17
Interventions	(0.23)	(0.31)	(0.76)	(0.90)	(0.11)	(0.21)
5. Control Past	0.47	0.37	1.53	1.43	0.03	-0.03
Stock Returns	(0.18)	(0.23)	(0.59)	(0.68)	(0.08)	(0.13)
6. Control Past	0.46	0.37	1.51	1.42	0.04	-0.01
10-Year Yield	(0.17)	(0.23)	(0.60)	(0.69)	(0.07)	(0.13)
7. Drop Around	0.37	0.28	1.12	0.91	0.02	-0.05
the Cutoff Changes	(0.16)	(0.21)	(0.44)	(0.48)	(0.06)	(0.12)

Table 1: Robustness

Notes: Table 1 shows robustness checks against various modifications to our benchmark specifications. Panels A and B show responses of stock prices and the JGB 10-year yield, respectively. In each panel, row 0 shows the baseline estimates. Row 1 considers bandwidth that is 50% less than the original one. Row 2 considers bandwidth that is 50% more than the original one. Row 3 considers a local polynomial regression of order 2 instead of 1. Row 4 controls the BoJ's stock purchases in the past two days. Row 5 controls stock market returns over the past two days. Row 6 controls changes in the 10-year yield over the past two days. Row 7 drops observations before and after one week around the cutoff changes. Same-day response indicates the changes in the outcome variable from 11AM to 3PM in the same day of the intervention. Next-day response indicates the changes in the outcome variable from 11AM of the day of the intervention to 9AM in the next day. Standard errors, which account for heteroskedasticity and autocorrelation, are reported in parenthesis.

(A) Placebo Test for Stock Response **(B)** Placebo Test for 10-year JGB Response

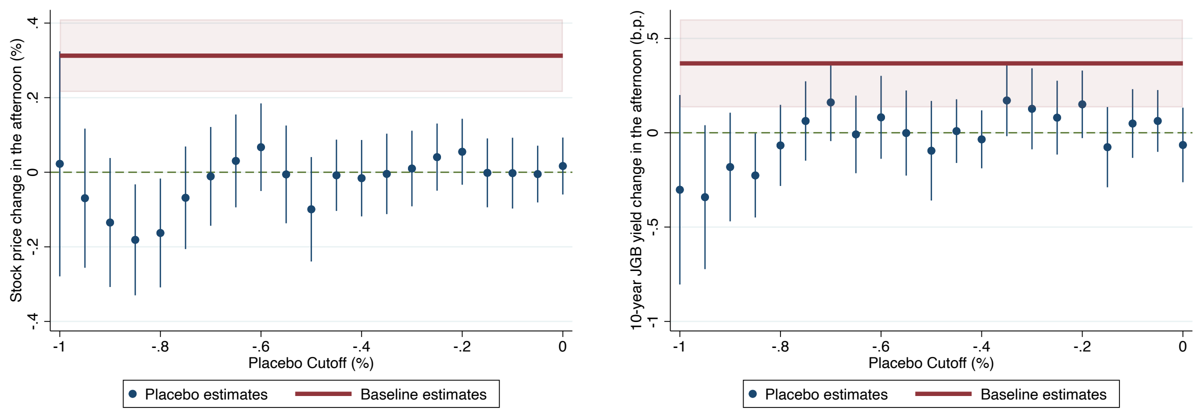

Figure 10: Placebo Tests

Notes: Figure 10A plots the estimates of $\gamma_{l,h}$ in equation (4) as the blue dots for each placebo cutoff, where the outcome variable is the stock price. The estimates are the response within the day of intervention (changes from 11AM to 3PM). We exclude the periods where the placebo cutoff coincides with the actual cutoff. The red line indicates our estimates using the actual cutoff. Figure 10B is analogous to Figure 10A, where the outcome variable is now the 10-year JGB yield. The line and the shaded area represent 90% confidence interval.

4.6 Other Discussions

We discuss several other issues. First, if the BoJ is selling the long-term government bonds at the same time as the stock purchases, then it is not surprising that the long-term interest rate rises in response to the stock purchases. However, during our sample periods, the BoJ has sold the government bonds only twice, March 24, 2017 and March 23, 2020, both of which are the periods after the yield curve control. Therefore, we can forcefully rule out the concern.

Second, one may wonder whether the difference in the amount of purchases across time periods might be driving the heterogeneity between before and after the yield curve control. Figure B.4 in the appendix shows that, while it is true that the absolute amount of purchases in each intervention is four times larger after the yield curve control than before, the growth is much less pronounced once expressed as a fraction of market capitalization. The average size of intervention as a fraction of market capitalization is slightly less than twice after the yield curve control than before.

The final issue concerns the interpretation of our results. As discussed by Krishnamurthy and Vissing-Jorgensen (2011), there are broadly two channels through which central bank asset purchases have an effect on asset prices. The first is the signaling channel. According to this channel, the central bank asset purchases have an effect because they send signals about the future policy stance of the central bank. The second channel is the liquidity channel. This channel operates through changing the aggregate demand and supply of assets. We believe our empirical results are likely driven by the liquidity channel rather than the signaling channel. The BoJ announces the target amount of stock purchases in each year in advance. Therefore, whether or not the BoJ purchases stocks today should not reveal the BoJ's future policy stance.

5 Organizing Theoretical Framework

We provide an organizing theoretical framework to provide structural interpretations of our empirical results. Through the lens of our model, we argue that the empirical estimates support a theory in which both stock and bond markets are substantially inelastic.

5.1 Setup

The only factor of production is capital. We assume that the supply of capital is fixed at K and

$$Y_t = A_t K,$$

where productivity A_t evolves stochastically and independently over time according to log-normally distributed growth

$$\ln(A_{t+1}/A_t) \sim N(g - \frac{1}{2}\sigma^2, \sigma^2),$$

where g is the mean growth rate and σ^2 is the variance of growth rate.

Households are divided into two parts: a consumption part and an investment part. The consumption part of the household makes consumption and saving decisions. The investment part of the household decides the amount to invest in stocks. Each part takes the other part's action, as given. This structure closely follows Gabaix and Koijen (2021), which makes our comparison to them easier and also helps simplify the exposition.

The consumption part of the household solves the following problem.

$$\begin{aligned}
&\max_{\{C_t, B_t^c\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t \exp(v_b(B_t/Y_t))) \\
&\text{s.t.} \quad C_t + B_t + S_t = W_t - F_t \\
&\qquad\quad W_{t+1} = R_t B_t + R_t^s S_t - T_{t+1},
\end{aligned}$$

where C_t is the consumption, $B_t \equiv B_t^i + B_t^c$ is the total bond holdings with the rate of return R_t, and W_t is the aggregate wealth at time t. Here, B_t^c denotes the bond-holding decisions made by the consumption part of the household, and B_t^i denotes the bond-holding decisions made by the investor part of the household. Central bank finances the cost of purchases through the lump-sum tax F_t, and it distributes ex-post profits through T_{t+1}. The stocks, S_t, with return R_t^s and the investor's bond holding decisions, B_t^i, are managed by the investor part of the household.

The preference term $v_b(\cdot)$ captures the preference for liquidity or safety as in Krishnamurthy and Vissing-Jorgensen (2012), with the properties $v_b' \geq 0$ and $v_b'' \leq 0$.

The investor part of the household chooses the portfolio to maximize the following objective function:[9]

$$\max_{S_t, B_t^i, W_{t+1}} \mathbb{E}_t \frac{[W_{t+1} \exp(-v_s(S_t/Y_t - \bar{s}))]^{1-\gamma}}{1-\gamma}.$$

The return from stock is given by

$$R^s_{t+1} = \frac{A_{t+1} + P_{t+1}}{P_t},$$

where P_t is the stock price. The term v_s is the adjustment cost that is similar in spirit to Gabaix and Koijen (2021). This can be broadly interpreted as any factors that prevent flexible adjustment in the portfolio, for example, inattention, inertia, or institutional mandate. We assume the investor has to incur adjustment costs when the portfolio deviates from the equilibrium without the central bank. Since $S_t = P_t K$ holds without the central bank, it follows that $\bar{s} \equiv P_t K / Y_t$. We impose $v_s' \geq 0$, $v_s'' \geq 0$, and $v_s'(0) = 0$. We assume the investor part of the household takes the b_t, the decision of the consumption part of the household as given.

The central bank's budget constraint is

$$S_t^{CB} + B_t^{CB} = R^s_{t-1} S^{CB}_{t-1} + R_{t-1} B^{CB}_{t-1} + T_t + F_t. \tag{5}$$

The equilibrium of this economy consists of prices $\{p_t, R_t\}$, quantities $\{c_t, a_t, s_t\}$, and central bank's policies $\{B_t^{CB}, S_t^{CB}, T_t, F_t\}$ such that (i) given $\{P_t, R_t, S_t\}$, $\{C_t, B_t\}$ solve the problem of households and mutual funds; (ii) given $\{P_t, R_t, B_t\}$, the investor part of the household optimally chooses $\{S_t\}$; and (ii) $\{B_t^{CB}, S_t^{CB}, T_t\}$ satisfies the central bank's budget constraint (5); and (iii) markets clear as follows:

$$\begin{aligned} C_t &= Y_t \\ B_t^i + B_t^c + B_t^{CB} &= 0 \\ S_t + S_t^{CB} &= P_t k. \end{aligned}$$

[9]The assumption that the investor solves myopic portfolio problem follows Gabaix and Koijen (2021). This assumption is not essential for any part of our analysis other than to make the comparison to Gabaix and Koijen (2021) easier.

5.2 Equilibrium without Central Bank

We first characterize the balanced growth path (BGP) of the equilibrium without the central bank. Here, we delegate the detailed derivation to Appendix C. The first order condition of the consumption part of the household combined with the market clearing conditions, $B_t = 0$ and $C_t = Y_t$, gives

$$1 = \beta R_t \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)} + v_b'(0), \tag{6}$$

which is the usual consumption Euler equation with the additional term capturing the demand for liquidity service. In solving the problem of the investor part of the household, we work with the following approximate portfolio problem as the time interval goes to zero, following Campbell and Viceira (2002).

$$\max_s \mathbb{E}(\ln(R_t^s) - \ln R)s + (1-\gamma)s^2 Var(\ln R_t^s) - v_s(sP_tK_t/Y_t - \bar{s}).$$

We conjecture (and verify) that the stock price is constant proportional to the productivity, $P_t = pA_t$. Taking the FOC of the above problem and imposing the market clearing, $s_t = 1$, we obtain

$$\left(g - \ln R + \ln \frac{1+p}{p}\right) = \gamma\sigma^2 + v_s'(0)p. \tag{7}$$

Equilibrium prices $\{p, R_t\}$ solve (6) and (7).

5.3 Central Bank Stock Purchases

We now study the effect of central bank stock purchases. To do so, we work with the log-linearized approximation of the no central bank equilibrium described above. We consider two cases: a case where interest rates flexibly adjusts and a case where interest rates do not adjust in response to the central bank's intervention.

5.3.1 Flexible Interest Rate Adjustment

In this case, we consider the following policy intervention. We assume the central bank permanently holds a fraction of stocks $ds^{CB} = dS_t^{CB}/P_tK > 0$ amount of stock financed by the bond issuance $dB_t^{CB} = -ds_t^{CB}P_tK$. The central bank's ex-post return is distributed through the lump-sum transfers: $T_t = -(R_{t-1}^s S_{t-1}^{CB} + R_{t-1}B_{t-1}^{CB})$.

The log-linearized Euler equation is

$$d\ln R = \kappa_b ds^{CB}, \tag{8}$$

where $\kappa_b \equiv \frac{-v_b''(0)}{1-v_b'(0)}p$. We refer to the parameter κ_b as the bond market inelasticity, and it captures how a one percentage point increase in the supply of bonds as a fraction of market capitalization affects the interest rate. The log-linearized asset pricing equation is

$$d\ln p = -\gamma_r d\ln R + \kappa_s ds^{CB} \tag{9}$$

where $\gamma_r \equiv \frac{1+p}{1+(1+p)pv_s'(0)} = \frac{d\ln p}{d\ln R}$ is the elasticity of stock price with respect to the interest rate, and $\kappa_s \equiv \frac{(1+p)p^2 v_s''(0)}{1+(1+p)pv_s'(0)}$ is the stock market inelasticity *holding the interest rate fixed.*

The following proposition summarizes the qualitative theoretical predictions:

Proposition 1. *Consider the small amount of stock purchases by the central bank described above.*

i. ***Elastic stock and bonds market.*** *If $v_b(\cdot) = v_s(\cdot) = 0$, then the central bank stock purchases are neutral:*

$$d\ln R = d\ln p = 0.$$

ii. ***Inelastic stock market and elastic bonds market.*** *If $v_s''(0) > 0$ and $v_b''(0) = 0$, then*

$$d\ln R = 0, \quad d\ln p > 0.$$

iii. ***Inelastic stock and bond market.*** *If $v_s''(0) > 0$ and $v_b''(0) < 0$, then*

$$d\ln R > 0$$

$$d\ln p \begin{cases} > 0 & \kappa_s > \gamma_r \kappa_b \\ = 0 & \kappa_s = \gamma_r \kappa_b \\ < 0 & \kappa_s < \gamma_r \kappa_b \end{cases}.$$

The first result is reminiscent of the neutrality result of the central bank portfolio by Wallace (1981). In a frcitionlessenvironment where Ricardian equivalence holds and the private agents are free to adjust their portfolios, any movements in the central bank portfolio are completely undone by private agents.

Figure 11 graphically illustrates the frictionless benchmark. The left and the right panels show the demand and supply curves for stocks and bonds, respectively. Since both assets are in fixed supply, the supply curves are fixed. In the frictionless economy, the demands are infinitely elastic, and as a result, demand curves are horizontal. As a result, a central bank stock purchase, which absorb the supply of stocks in the market by providing additional supply of bonds to the market, have no consequence on the stock prices or interest rates.

The second result is reminiscent of the stock market inelasticity hypothesis recently proposed by Gabaix and Koijen (2021). When the stock market is inelastic, the central bank stock purchases cannot be undone by private agents. This implies that the demand for stock rises,

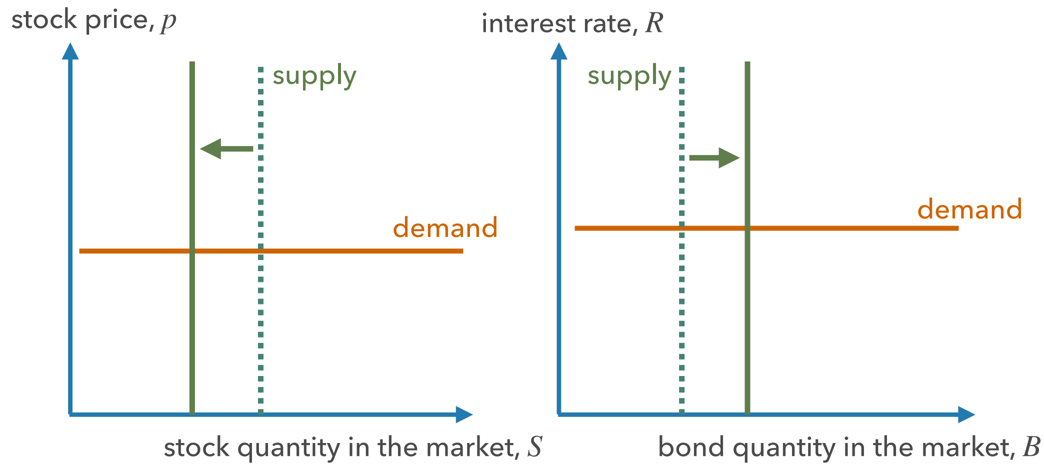

Figure 11: Frictionless Benchmark

which drives up the stock price. In contrast, with an elastic bonds market, the interest rate is pinned down by the consumption Euler equation, which is unaffected by the central bank portfolio.

The third result is our focus, and it provides a much more nuanced view than the previous two cases. When both bonds and stock markets are inelastic, the interest rate unambiguously rises, and the effect on stock prices is ambiguous. The interest rate rises because as the central bank supplies more bonds, the households value less of it, which in turn puts downward pressure on bond prices. This rise in the interest rate can counteract the rise in the stock price. Three sufficient statistics, κ_s, κ_b, and γ_r, govern which of these forces dominate in a intuitive manner. In fact, it boils down to the comparison of stock market inelasticity, κ_s, and the multiplicative of bonds market inelasticity and the interest rate sensitivity of stock prices, $\kappa_b\gamma_r$. When the stock market is more inelastic (high κ_s), the stock price is more likely to rise. When the bonds market is relatively inelastic (high κ_b) and the sensitivity of stock prices to interest rates is high (high γ_r), stock price is more likely to fall.

Figure 11 graphically illustrates the case where both stock and bond markets are inelastic. Unlike the frictionless benchmark, the demand curve for stock is downward sloping with respect to price, and the demand curve for bond is upward sloping with respect to interest rate. Moreover, the demand for stock negatively depends on the interest rate. In this economy, central bank stock purchases will have a direct impact of increasing the stock price and interest rate, as can been seen as the outcome of the shift in supply curve. The increase in in interest rate, in turn, reduces the demand for stock. Therefore, the net effect on stock price is ambiguous.

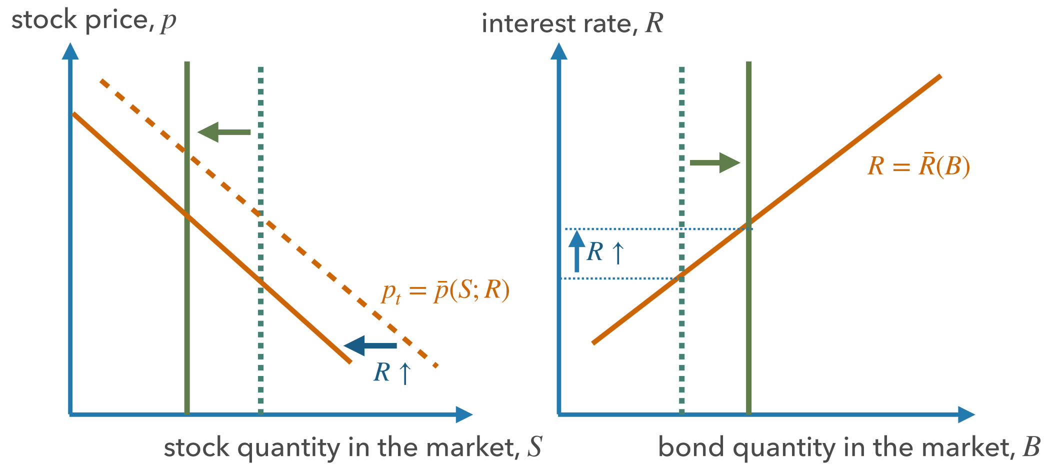

Figure 12: Inelastic Stock and Bond market

5.3.2 Fixed Interest Rate

Now we consider the central bank stock purchases financed through lump-sum taxes: $dS_t^{CB}/P_tK \equiv ds^{CB} > 0$ with $dB_t^{CB} = 0$ and $dT_t = dS_t^{CB}$ for $t = 0$ and $dT_t = dS_t^{CB} - R_{t-1}^s S_{t-1}^{CB}$ for $t \geq 1$. In this economy, this is the only way that the central bank is able to maintain a constant real interest rate when it buys stocks. We intend to mimic the central bank stock purchases under the yield curve control.

In this case, we have

$$\begin{aligned} d\ln p_t &= \kappa_s ds^{CB} \geq 0 \qquad (10) \\ d\ln R_t &= 0. \end{aligned}$$

The fact that the interest rate does not respond stems from the fact that the central bank does not issue bonds to finance the purchases of stocks, which implies the household Euler equation is unaffected. Because the interest rate is fixed, the stock price always (weakly) increases. Figure 13 graphically illsutrates the case with fixed interest rate. Since the interest rate adjustment is absent, there is downward shift in the demand curve for stock. As a result, the stock price robustly rises. Qualitatively, this prediction also corresponds to the case where bond market is perfectly elastic.

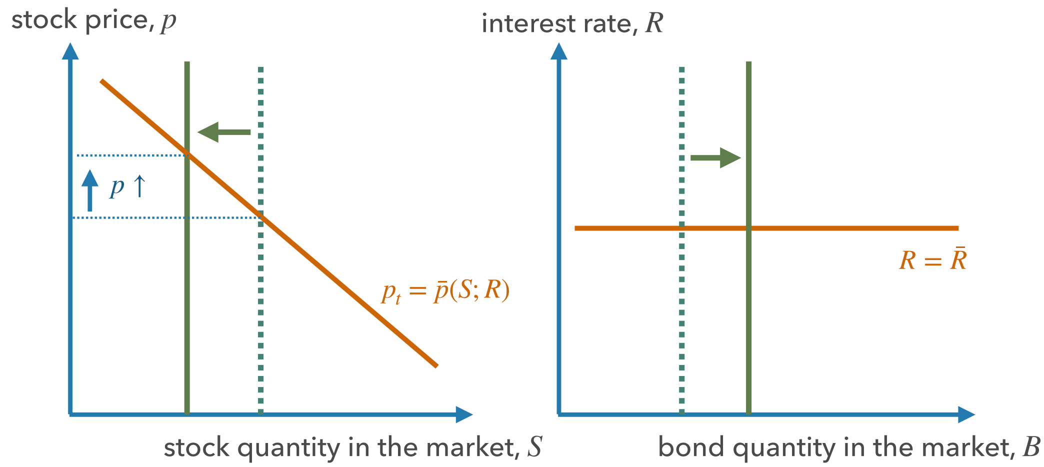

Figure 13: Fixed Interest Rate

5.4 Mapping to the Empirical Analysis and Identification of the Structural Model

We now connect our structural model to our empirical analysis. Note that equations (9) and (8) are exactly the equations that we estimated in the empirical sections. Taking our benchmark estimates for the next day (0th row of Table 1), we have

$$\kappa_b = 1.41 \tag{11}$$

$$\gamma_r \kappa_b + \kappa_s \approx 0, \tag{12}$$

where we set the latter to zero, since our estimates are noisy and indistinguishable from zero. In the post-YCC sample, assuming YCC corresponds to the fixed interest rate, equation (10) corresponds to our estimating equation for the stock price. Therefore, we obtain

$$\kappa_s = 22. \tag{13}$$

Gabaix and Koijen (2021) report empirical estimates of stock market inelasticity of 5. Other studies (Da et al., 2018; Hartzmark and Solomon, 2021; Li et al., 2021) estimate somewhere between 1.5 to 6. While our estimates of stock market inelasticity holding the interest rate fixed, κ_s, is several times higher than their estimates, the stock market elasticity when interest rate can freely move, $\gamma_r \kappa_b + \kappa_s$, is lower than theirs (our point estimates are even negative). The difference could come from either the difference in identification strategy or the difference in the underlying macroeconomic environment (e.g., persistently low interest rate environment in Japan). However, the robust message of our paper is to underscore the importance of jointly taking into account stock market and bond market inelasticity.

We close this section by providing an over-identification test. From equations (11), (12), and (13), one can back out the stock price elasticity with respect to the interest rate as

$$\gamma_r = -15.6.$$

This is well in line with existing empirical estimates in the context of Japan. Kubota and Shintani (2022a) use high-frequency identification around monetary policy announcements by the BoJ. They find that unanticipated 1 percentage point monetary policy tightening results in a 10% to 16% drop in stock prices.[10] Therefore, our empirical estimates pass the over-identification test, which renders further credibility.

6 Conclusion

By exploiting the unique feature of Bank of Japan's policy rule, we trace the aggregate impact of stock market purchases. Taken together, our empirical evidence provides support for a theory in which both stock and bond markets are substantially inelastic. We believe our results will prove useful in designing quantitative easting policies around the globe and to understanding the sources of financial market fluctuations. The natural next step is to assess how the central bank stock purchases have transmitted to the real economy, which we leave for future research.

[10] Kubota and Shintani (2022b) extend the analysis to examine the effect for longer horizons using VAR with external instruments. They show that a 1% monetary policy tightening results in around a 25% drop in stock prices after one year. All these estimates are substantially higher (in absolute terms) than the estimates from similar settings using the US data (e.g., Bernanke and Kuttner, 2005), possibly reflecting the persistently low interest rate environment in Japan.

Appendix

A Empirical Appendix

A.1 Local Nonlinear Impulse Response Function

In this section, we allow nonlinearity in the impulse response and show that our estimands can be interpreted as dynamic "local average treatment effect" in the spirit of Angrist and Imbens (1995).

We first define a potential outcome framework in our context following Rambachan and Shephard (2021). For each $t \geq 1$, the BoJ decides the amount of ETF purchases ETF_t and let us denote $ETF_{1:t} \equiv (ETF_1, \cdots, ETF_t)$. Let $w_{1:t} \equiv (w_1, \cdots, w_t)$ be a potential assignment path up to t where $w_t \in [0, \bar{w}]$ for all t. Associated with this potential assignment path, potential outcome at day $t+l$ time h is $Y_{t+l,h}(w_{1:t+l})$.[11] Note that for any different assignment paths, there exist different outcome paths but we only obeserve $Y_{t+l,h}(ETF_{1:t+l})$. For any day $t+l$ time h, let us denote

$$Y_{t+l,h}(w) \equiv Y_{t+l,h}(ETF_{1:t-1}, \underbrace{w}_{t-th}, ETF_{t+1:t+l}).$$

Using this notation, the observed outcome can be denoted as $Y_{t+l,h}(ETF_t)$ by definition. Now, let the outcome be$\Delta y_{t+l,h} \equiv y_{t+l,h} - y_{t,0}$, which is the change in variable y (e.g. stock prices and interest rates) from the end price of the morning session to at day t to time h at day $t+l$,

$$\Delta y_{t+l,h} = Y_{t+l,h}(ETF_t). \tag{14}$$

We assume that the BoJ's ETF purchasing policy rule takes the following form, in which the amount of ETF purchase at time t, ETF_t, is given by

$$ETF_t = ETF_{-,t}(\Delta p_t)\mathbb{I}(\Delta p_t < c_t) + ETF_{+,t}(\Delta p_t)\mathbb{I}(\Delta p_t \geq c_t), \tag{15}$$

where Δp_t is the log-changes in the TOPIX value in the morning, c_t is the cut-off, and $ETF_{-,t}$ and $ETF_{+,t}$ are random functions of Δp_t which represent different policy rules depending on whether Δp_t is above or below the cutoff at time t. The following assumptions guarantee that our estimands identify the dynamic local average treatment effect.

[11] We assume that the potential outcome depends only on past and contemporaneous assignments. Rambachan and Shephard (2021) called this assumption Non-antiripating potential outcomes.

Assumption 1. *(i)* $Y_{t+l,h}(w)$ *is bounded and continuously differentiable in* $w \in [0, \bar{w}]$ *with probability one and, (ii)* $ETF_{-,t}(\Delta p)$ *and* $ETF_{+,t}(\Delta p)$ *are bounded and continuous at* c_t *with probability one.*

Assumption 2 (Monotonicity). $ETF_{-,t}(c_t) \geq ETF_{+,t}(c_t)$ *with probability one.*

Assumption 3 (Relevance). $\int \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t)dw > 0$.

Assumption 4 (Local Independence). *For each* $t + l$ *and* h*, there exists a neighborhood* $N_{t+l,h}$ *of* c_t *such that* $\Delta p_t \perp (\{Y_{t+l,h}(w)\}_w, ETF_{-,t}(c_t), ETF_{+,t}(c_t))|\Delta p_t \in N_{t+l,h}$.

Theorem 1. *If Assumptions 1-4 hold, then*

$$\frac{\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p]}{\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p]}$$
$$= \int \mathbb{E}[\frac{\partial Y_{t+l,h}(w)}{\partial w}|\Delta p_t = c_t, ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)]\bar{\omega} dw,$$

where $\bar{\omega} = \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t) / \int \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t)dw$.

Proof. First, observe that

$$\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p] = \lim_{\Delta p \downarrow c_t} \mathbb{E}[Y_{t+l,h}(ETF_{-,t}(\Delta p))|\Delta p_t = \Delta p]$$
$$= \mathbb{E}[Y_{t+l,h}(ETF_{-,t}(c_t))|\Delta p_t = c_t],$$

where the second equality follows from Assumption 1 and 4. Therefore,

$$\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p]$$
$$= \mathbb{E}[Y_{t+l,h}(ETF_{-,t}(c_t)) - Y_{t+l,h}(ETF_{+,t}(c_t))|\Delta p_t = c_t]$$
$$= \mathbb{E}[\int \frac{\partial Y_{t+l,h}(w)}{\partial w} \mathbb{I}\{ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)\} dw|\Delta p_t = c_t]$$
$$= \int \mathbb{E}[\frac{\partial Y_{t+l,h}(w)}{\partial w}|\Delta p_t = c_t, ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)] \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t)dw,$$

where second equality follows from Assumptions 1and 3, and the third equality follows from

1. Similarly,

$$\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p] = \int \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t)dw,$$

and Assumption 2 guarantees that the denominator is positive. Combining these, we have the stated result. □

Since $Y_{t+l,h}(w) \equiv Y_{t+l,h}(ETF_{1:t-1}, w, ETF_{t+1:t+l})$, the local independence assumption requires that falling below the cutoff at day t is not correlated with the future or past ETF purchases. We test this in Appendix A.4.

A.2 Details on Cutoff Estimation

We first split the sample based on six announcements by the BoJ that publicized changes in the target amount of ETF purchases on April 4, 2013, October 31, 2014, December 18, 2015, July 29, 2016, July 31, 2018, and March 16, 2020. We then divide each sample based on whether TOPIX value falls below zero for the past two consecutive days. For the case with consecutive drops in the past two days, we further split on April 1, 2019, for the reason that we describe below.

In each sample split, we proceed as follows. We take grid points for the cutoff candidates from -1% to 0% with 0.05% interval, $\mathbb{C} = \{-1.0\%, -0.95\%, \ldots, -0.05\%, 0.0\%\}$. For each of $c \in \mathbb{C}$, we estimate the following linear probability model separately on both sides of the candidate cutoff, c:

$$\Pr_{-,t}(ETF_t > 0|\Delta p_t) = \begin{cases} \alpha_- + \beta_- \Delta p_t & \text{for } \Delta p_t \in [c-k, c] \\ \alpha_+ + \beta_+ \Delta p_t & \text{for } \Delta p_t \in [c, c+k] \end{cases}, \tag{16}$$

where we take the bandwidth to be 1% around the cutoff, $k = 1\%$. Given the estimates, we can compute the jump around the cutoff as follows:

$$J_t(c) \equiv \lim_{\Delta p \uparrow \bar{c}} \widehat{\Pr}_t(ETF_t > 0|\Delta p) - \lim_{\Delta p \downarrow \bar{c}} \widehat{\Pr}_t(ETF_t > 0|\Delta p),$$

where $\widehat{\Pr}_t$ denote the fitted value of equation (16). We select the cutoff that maximizes square of the jump:

$$c_t^* \in \arg\max_{c \in \mathbb{C}} J_t^2(c).$$

Whenever there is a tie, we choose the largest cutoff.

Table B.2 shows the estimated cutoff, and Table B.3 shows the discontinuity in the probability of Bank of Japan's intervention around the estimated cutoff. As argued in the main text, the estimated cutoffs align well with what is commonly argued among media. The discontinuity around the cutoff is always over 50%, is often over 80%, and they are highly statistically significant. We made a choice to split the sample with consecutive drops in the past two days at April 1, 2019, because there was a apparent change in the cutoff around this period. If we do not split the sample at this point in time, the resulting discontinuity is -0.744 . If we split the sample, the discontinuity is -1.000 in the first half, and it is -0.853 in the second half of the sample. This choice does not materially affect any of our empirical results.

Figure B.1 graphically displays the discontinuity in the probability of intervention for each period. While the magnitude of discontinuity is more apparent in the beginning and the end of the sample period, the sharp discontinuity shows up in all subsamples.

A.3 Manipulation Test

A typical concern in regression discontinuity-based identification strategies is manipulation (McCrary, 2008). We first note that this concern is unlikely in our context since there is little room for investors to precisely manipulate the stock price index. Having said this, we formally test the presence of manipulation by examining the continuity of density function of TOPIX changes in the morning. We estimate the density function using the local polynomial density estimator by Cattaneo et al. (2020) and test the presence of discontinuity around our estimated cutoff.

Figure B.2 shows the estimated density and histogram, and Table B.4 reports the estimates and test statistics for discontinuity. While there is a small mass on the right side of the cutoff, the p-value of testing the discontinuity is 0.447. Therefore, there is no statistical evidence of manipulation.

A.4 (Dis)continuity of ETF Purchases across Days

In this section, we argue that the effects we are identifying are the effects of a one-time shock of ETF purchases. As discussed in A.1, $y_{t+l,h}$ is clearly affected by the BoJ's ETF purchases up to l days later. Therefore, if falling below the cutoff today is correlated with the future and past purchases, our empirical estimates cannot be interpreted as the causal effect of one-time BoJ ETF purchases (Rambachan and Shephard, 2021). In order to address this concern, we estimate the discontinuity in the amount of ETF purchases around the cutoff across days. Formally, we estimate the following term,

$$\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_{t+l} | \Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_{t+l} | \Delta p_t = \Delta p]. \tag{17}$$

Figure B.3 shows the estimates of discontinuity of the amount of ETF purchases at date $t + l$ around $\Delta p_t = c_t$. Reassuringly, we find significant discontinuity only at $l = 0$. Therefore, our identified effects are the causal effects of one-time the BoJ's ETF purchases and are not contaminated by the future or past ETF purchases.

B Additional Tables and Figures

Table B.1: Major Announcements by the BoJ

Date	Annoucnement
October 28, 2010	Intention to purchase 450 billion yen of ETFs
October 30, 2012	Intention to purchase 2.1 trillion yen of ETFs annually
October 31, 2014	Annual purchase target increased to 3 trillion yen
December 18, 2015	Annual purchases target increased to 3.3 trillion yen
July 29, 2016	Annual purchases target increased to 6 trillion yen
March 16, 2020	Annual purchases target increased to 12 trillion yen

Notes: Table B.1 shows the six major announcements by the BoJ regarding the target ETF purchase amounts. Source: Fukuda and Tanaka (2022).

Table B.2: Estimated Cutoff

No Consecutive Drops		Consecutive Drops	
Period	Cutoff	Period	Cutoff
2010/12/15 - 2013/04/03	-1%	2010/12/15 - 2013/04/03	-1%
2013/04/04 - 2014/10/30	-0.35%	2013/04/04 - 2014/10/30	0%
2014/10/31 - 2015/12/17	-0.15%	2014/10/31 - 2015/12/17	0%
2015/12/18 - 2016/07/28	-0.4%	2015/12/18 - 2016/07/28	0%
2016/07/29 - 2018/07/30	-0.3%	2016/07/29 - 2018/07/30	0%
2018/07/31 - 2020/03/15	-0.5%	2018/07/31 - 2020/03/15	-0.25%
2020/03/16 - 2020/12/31	-0.5%	2020/03/16 - 2020/12/31	-0.25%

Notes: Table B.2 shows the estimated cutoff for each of the subsample.

Table B.3: Discontinuity in Probability of Intervention around the Estimated Cutoff

	No Consecutive Drops			Consecutive Drops		
	Discontinuity	Sample size		Discontinuity	Sample size	
	estimates	Left	Right	estimates	Left	Right
2010/12/15 - 2013/04/03	-1.011	43	158	-1.000	15	47
	(0.012)			(0.000)		
2013/04/04 - 2014/10/30	-0.576	60	146	-0.931	25	30
	(0.100)			(0.072)		
2014/10/31 - 2015/12/17	-0.683	72	98	-1.122	11	17
	(0.099)			(0.117)		
2015/12/18 - 2016/07/28	-0.811	20	36	-1.000	8	11
	(0.119)			(0.000)		
2016/07/29 - 2018/07/30	-0.604	78	243	-0.945	40	33
	(0.069)			(0.053)		
2018/07/31 - 2020/03/15	-0.978	49	163	-0.744	30	34
	(0.022)			(0.130)		
2020/03/16 - 2020/12/31	-0.930	29	71	-0.985	13	14
	(0.070)			(0.022)		

Notes: Table B.3 shows the discontinuity in the probability of the BoJ intervention around the estimated cutoff. We estimate the discontinuity using the local linear regression with bandwidth 1% around the cutoff and uniform kernel. The standard errors are reported in parenthesis.

Table B.4: Density Discontinuity Test

	Density estimates		Discontinuity test	
	Left	Right	Difference	p-value
	0.423	0.506	0.083	0.373
	(0.065)	(0.067)	(0.093)	
Sample size	667	1790		
Bandwidth	0.512	0.512		
Effective sample size	358	719		

Notes: Table B.4 reports the density estimates on the left and the right of cutoff and test statistics for the discontinuity test. We use the local polynomial density estimator by Cattaneo et al. (2020) with order 2. Robust standard errors are reported in parenthesis.

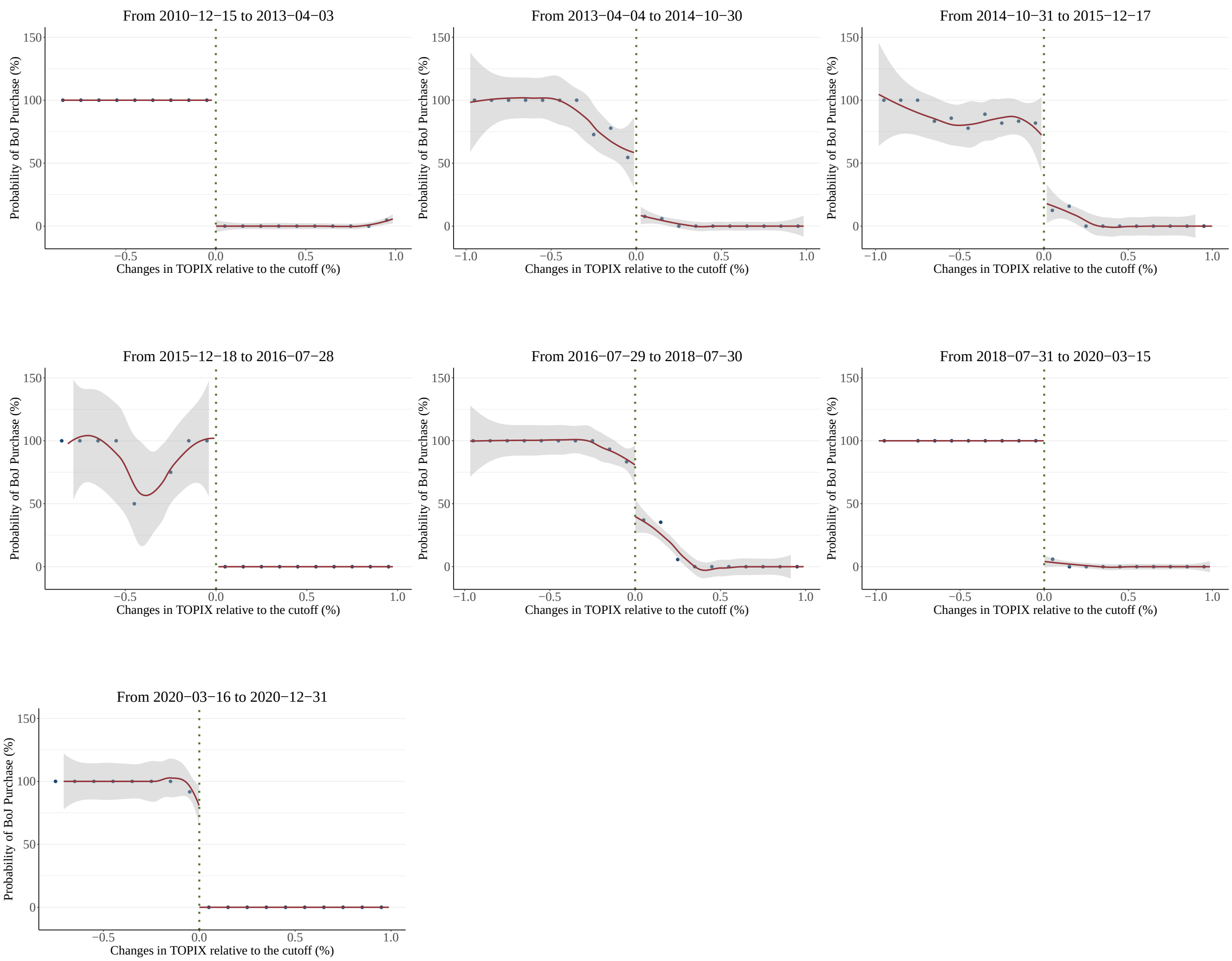

Figure B.1: Discontinuity in the Probability of the BoJ Intervention for each Period

Notes: Figure B.1 shows the discontinuity in the probability of the BoJ intervention around the estimated cutoff for each period. The blue scatter plot is the binned scatter plot with bin width 0.1%, and the red line indicates the LOESS fit with shaded gray area being the 95% confidence interval.

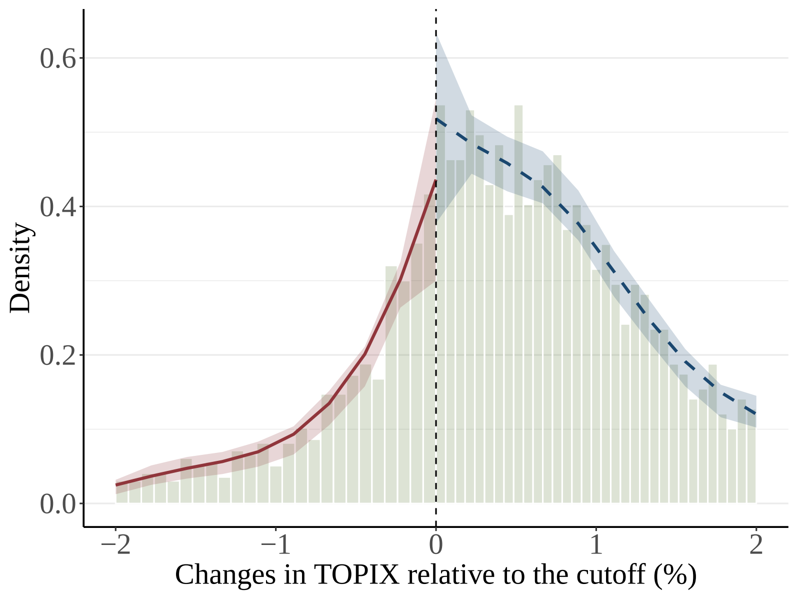

Figure B.2: Density around the cutoff
Notes: Figure B.2 shows the histogram and the density of changes in TOPIX relative to the cutoff. The shaded area is 95% confidence interval. We use the local polynomial density estimator by Cattaneo et al. (2020) with order 2.

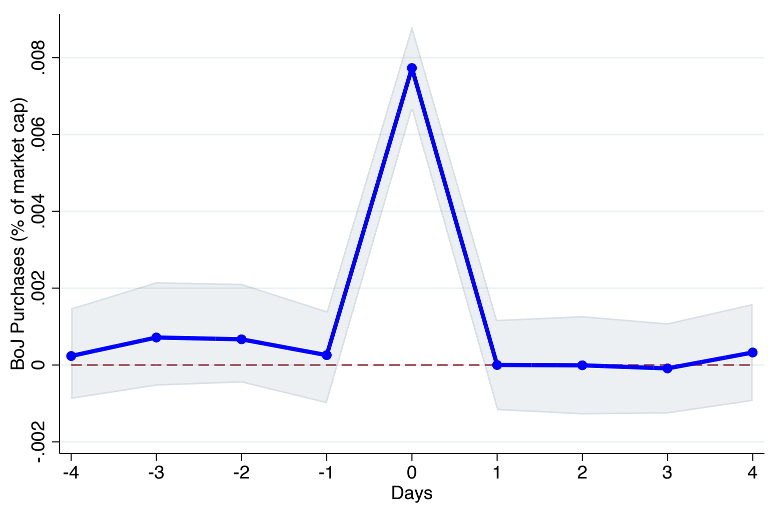

Figure B.3: (Dis)continuity of ETF Purchases across Days
Notes: Figure B.3 shows the estimates of (17) across days. The shaded areas represent 90% confidence intervals, which accounts for heteroskedasticity and autocorrelation.

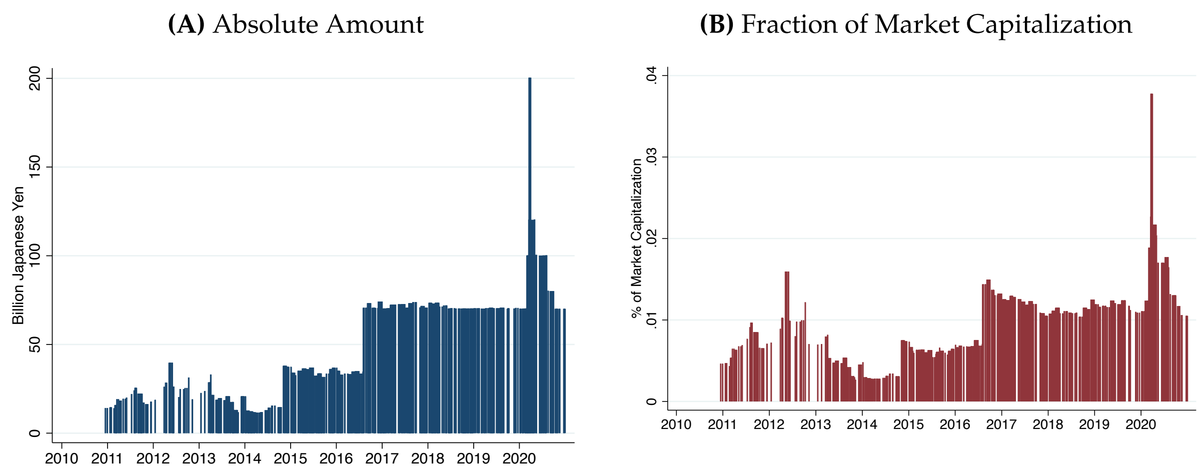

Figure B.4: The Amount of the BoJ Purchases

Notes: Figure B.4 plots the amount of stock purchases by the BoJ in each intervention. Figure B.4A shows the absolute amount of purchases in billion Japanese Yen (approximately 10 million US dollars). B.4B express it as a fraction of market capitalization.

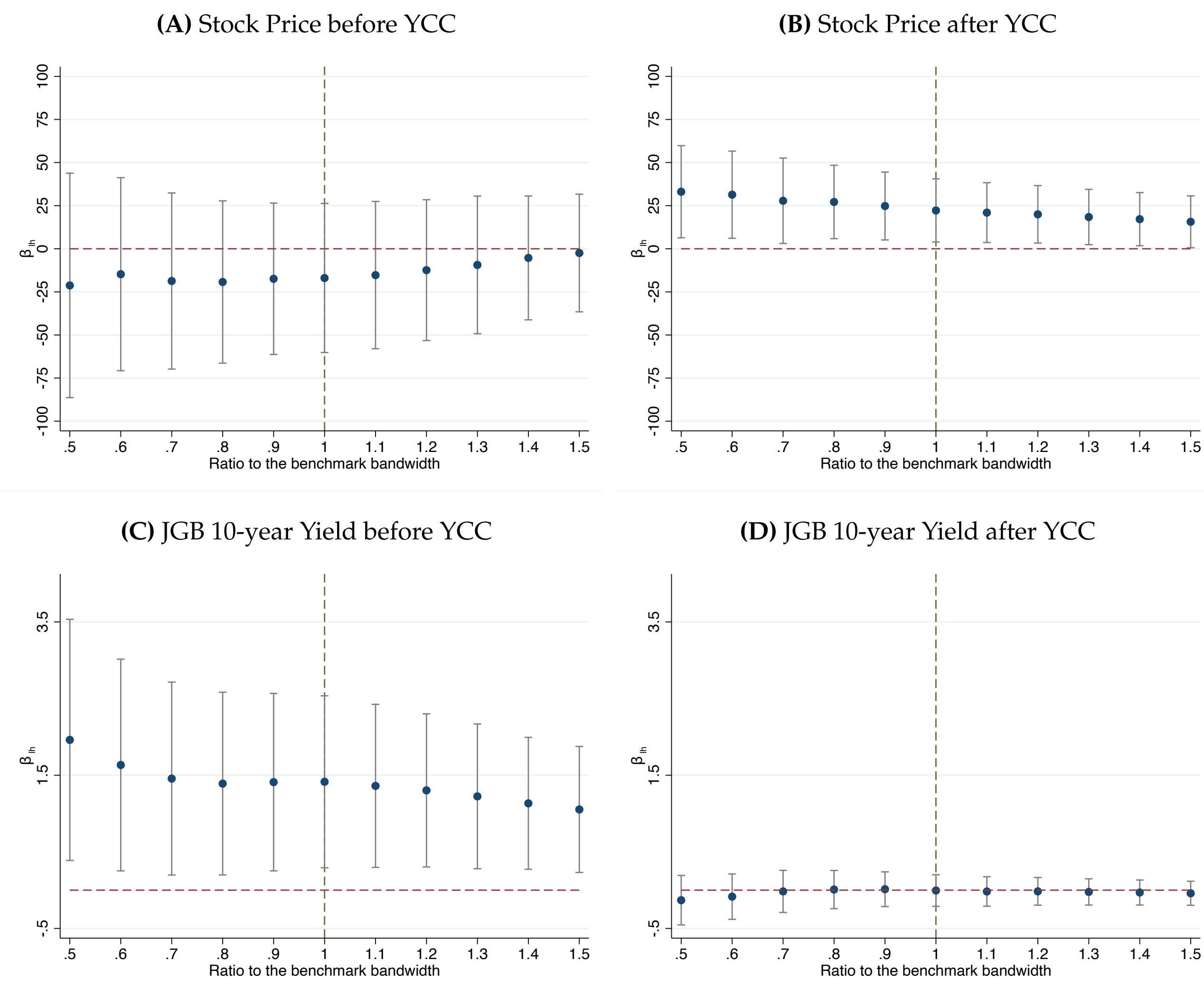

Figure B.5: Robustness to Bandwidth Selection

Notes: Figure B.5 shows the robustness of our estimates with respect to the size of the bandwidth. Each dot represents the point estimates of the response from 11AM of the intervention day to 9AM on the next day. The vertical line represents the 90% confidence interval, which accounts for heteroskedasticity and autocorrelation. The dashed green line is the optimal bandwidth proposed by Calonico, Cattaneo, and Titiunik (2014), which is our benchmark. Figures B.5A and B.5B show the response of stock price before and after YCC, respectively. Figures B.5C and B.5D show the response of 10-year JGB yield before and after YCC, respectively.

C Details on Theoretical Model

The Lagraingian of the consumption part of the household problem is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(C_t \exp(v_b(B_t/Y_t)) - \lambda_t \left[C_t + B_t^c + B_t^i + S_t - R_{t-1}(B_{t-1}^c + B_{t-1}^i) - R_{t-1}^s S_{t-1} + T_t \right] \right),$$

where λ_t is the Lagrangian multiplier for the budget constraint. The first order condition with respect to C_t and B_t^c are

$$u'(C_t \exp(v_b(B_t/Y_t)) \exp(v_b(B_t/Y_t)) = \lambda_t$$
$$u'(C_t \exp(v_b(B_t/Y_t)) C_t \exp(v_b(B_t/Y_t)) v_b'(B_t/Y_t) \frac{1}{Y_t} - \lambda_t = -\beta R_t \lambda_{t+1}.$$

We can impose market clearing, $C_t = Y_t$, and using the fact that $b = B_t/Y_t$ is constant along the balanced growth, and setting $v_b(b) = 0$,

$$u'(Y_t) - u'(Y_t) v_b'(b) = \beta R \mathbb{E}_t u'(Y_{t+1}). \tag{18}$$

Suppose the central bank holds a constant fraction of market portfolio, $s^{CB} \equiv S_t^{CB}/P_t K$, and issues the same amount of debt $B_t^{CB} = -s^{CB} P_t K$. Then, the bond market clearing implies $b = s^{CB} P_t K / Y_t = s^{CB} p$ where $p = P_t / A_t$. Substituting into 18, we obtain

$$1 - v_b'(s^{CB} p) = \beta R \mathbb{E}_t \frac{u'(Y_{t+1})}{u'(Y_t)}.$$

Log-linearizing around $s^{CB} = 0$, we obtain

$$d \ln R = \frac{-v_b''(0) p}{1 - v_b'(0)} ds^{CB}.$$

When the central bank does not issue bonds, it is immediate to see $d \ln R = 0$.

The investor part of the household solves

$$\max_{s_t, W_{t+1}} \mathbb{E}_t \frac{[W_{t+1} \exp(-v_s(s_t(W_t - C_t)/Y_t - \bar{s}))]^{1-\gamma}}{1-\gamma}$$
$$W_{t+1} = (R_t^s s_t + R_t(1 - s_t))(W_t - C_t) - T_{t+1},$$

where s_t is the portfolio share on stocks. As central bank rebates back the losses, we have $T_{t+1} = -(R_t^s - R) s^{CB} P_t K$. From the market clearing, $W_t - C_t = P_t K$. Substituting these conditions, we can rewrite as follows.

$$\max_{s_t, W_{t+1}} \mathbb{E}_t \frac{[W_{t+1} \exp(-v_s(s_t p - \bar{s}))]^{1-\gamma}}{1-\gamma} \tag{19}$$
$$W_{t+1} = \left(R_t^s (s_t + s^{CB}) + R_t (1 - s_t - s^{CB}) \right) P_t K_t.$$

Following Campbell and Viceira (2002), we assume investors solve an approximate version of the above portfolio problem in which log portfolio return is normally distributed. Taking log of the objective function,

$$\max_s \mathbb{E}_t \ln R_t^p + \frac{1}{2}(1-\gamma)\mathrm{Var}_t \left(\ln R_t^p \right) - v_s(s_t p - \bar{s}),$$

where $R_t^p \equiv \left(R_t^s (s_t + s^{CB}) + R_t(1 - s_t - s^{CB}) \right)$ is the portfolio return, where

$$\mathbb{E}_t \ln R_t^p \approx (1 - s_t - s^{CB}) \ln R + (s_t + s^{CB})(g + \ln \frac{1+p}{p} - \frac{1}{2}\sigma^2) + \frac{1}{2}(s_t + s^{CB})(1 - s_t - s^{CB})\sigma^2$$
$$\mathrm{Var}_t \left(\ln R_t^p \right) \approx (s_t + s^{CB})^2 \sigma^2.$$

Therefore the problem reduces to

$$\max_{s_t} (s_t + s^{CB})(g + \ln \frac{1+p}{p} - \ln R) - \frac{\gamma}{2}(s_t + s^{CB})^2 \sigma^2 - v_s(s_t p - \bar{s})$$

The first order condition gives

$$(g + \ln \frac{1+p}{p} - \ln R) - \gamma\sigma^2 = p v_s'(-s^{CB} p),$$

where we used $s = 1 - s^{CB}$ and $\bar{s} \equiv p$. Log-linearizing the above equation around $s^{CB} = 0$, we obtain

$$d \ln p = -\frac{(1+p)}{1 + (1+p) v_s'(0)} d \ln R + \frac{(1+p) p^2 v_s''(0)}{1 + (1+p) v_s'(0)} d s^{CB}$$

Now we characterize the investor's portfolio problem when the central bank only buys stocks so as to keep the interest rate fixed. The central bank purchases the constant share of market capitalization, $S_t^{CB} = s^{CB} P_t K_t$, financed with $F_t = S_t^{CB}$. The central bank rebates back the return so that $T_{t+1} = R_t^s S_t^{CB}$. The investor's problem is

$$\max_{S_t, B_t, W_{t+1}} \mathbb{E}_t \frac{[W_{t+1} \exp(-v_s(S_t / Y_t - \bar{s}))]^{1-\gamma}}{1-\gamma}$$
$$W_{t+1} = R_t^s S_t + R_t B_t + R_t^s S_t^{CB}.$$

Because $S_t^{CB} + B_t + S_t = P_t K$, we can rewrite the above problem as

$$\max_{s_t, W_{t+1}} \mathbb{E}_t \frac{[W_{t+1} \exp(-v_s(s_t p - \bar{s}))]^{1-\gamma}}{1-\gamma}$$
$$W_{t+1} = \left[R_t^s (s_t + s_t^{CB}) + R_t (1 - s_t - s_t^{CB}) \right] P_t K.$$

At this point, the problem is equivalent to (19). Therefore we have the exactly the same log-linearized equation:

$$d \ln p = -\frac{(1+p)}{1 + (1+p) v_s'(0)} d \ln R + \frac{(1+p) p^2 v_s''(0)}{1 + (1+p) v_s'(0)} ds^{CB},$$

with $d \ln R = 0$.

References

ADACHI, K., K. HIRAKI, AND T. KITAMURA (2021): "Supplementary Paper Series for the "Assessment" (1): The Effects of the Bank of Japan's ETF Purchases on Risk Premia in the Stock Markets," Tech. Rep. 21-E-3, Bank of Japan.

ANGRIST, J. D. AND G. W. IMBENS (1995): "Two-Stage Least Squares Estimation of Average Causal Effects in Models with Variable Treatment Intensity," *Journal of the American Statistical Association*, 90, 431–442.

ANGRIST, J. D., Ò. JORDÀ, AND G. M. KUERSTEINER (2018): "Semiparametric Estimates of Monetary Policy Effects: String Theory Revisited," *Journal of Business & Economic Statistics*, 36, 371–387.

BARBON, A. AND V. GIANINAZZI (2019): "Quantitative Easing and Equity Prices: Evidence from the ETF Program of the Bank of Japan," *The Review of Asset Pricing Studies*, 9, 210–255.

BERNANKE, B. S. AND K. N. KUTTNER (2005): "What Explains the Stock Market's Reaction to Federal Reserve Policy?" *The Journal of Finance*, 60, 1221–1257.

CALONICO, S., M. D. CATTANEO, AND R. TITIUNIK (2014): "Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs," *Econometrica : journal of the Econometric Society*, 82, 2295–2326.

CAMPBELL, J. Y. AND L. M. VICEIRA (2002): *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*, Clarendon Lectures in Economics, Oxford: Oxford University Press.

CATTANEO, M. D., M. JANSSON, AND X. MA (2020): "Simple Local Polynomial Density Estimators," *Journal of the American Statistical Association*, 115, 1449–1455.

CHAROENWONG, B., R. MORCK, AND Y. WIWATTANAKANTANG (2021): "Bank of japan equity purchases: The (non-) effects of extreme quantitative easing," *Review of Finance*, 25, 713–743.

CHODOROW-REICH, G. (2014): "Effects of Unconventional Monetary Policy on Financial Institutions," .

CHUNG, J.-C. (2020): "The Bank of Japan's Exchange-Traded Fund Purchase Program and Its Impact on Daily Stock Market Returns," 53.

COCHRANE, J. H. AND M. PIAZZESI (2002): "The Fed and Interest Rates - A High-Frequency Identification," *American Economic Review*, 92, 90–95.

DA, Z., B. LARRAIN, C. SIALM, AND J. TESSADA (2018): "Destabilizing Financial Advice: Evidence from Pension Fund Reallocations," *The Review of Financial Studies*, 31, 3720–3755.

FUKUDA, S.-I. AND M. TANAKA (2022): "The Effects of Large-scale Equity Purchases during the Coronavirus Pandemic," 30.

GABAIX, X. AND R. S. J. KOIJEN (2021): "In Search of the Origins of Financial Fluctuations: The Inelastic Markets Hypothesis," SSRN Scholarly Paper ID 3686935, Social Science Research Network, Rochester, NY.

GORODNICHENKO, Y. AND W. RAY (2017): "The effects of quantitative easing: Taking a cue from treasury auctions," Tech. rep., National Bureau of Economic Research.

HAHN, J., P. TODD, AND W. VAN DER KLAAUW (2001): "Identification and Estimation of Treatment Effects with a Regression-Discontinuity Design," *Econometrica : journal of the Econometric Society*, 69, 201–209.

HARADA, K. AND T. OKIMOTO (2021): "The BOJ's ETF Purchases and Its Effects on Nikkei 225 Stocks," *International Review of Financial Analysis*, 77, 101826.

HARTZMARK, S. M. AND D. H. SOLOMON (2021): "Predictable Price Pressure," SSRN Scholarly Paper 3853096, Social Science Research Network, Rochester, NY.

IMBENS, G. W. AND T. LEMIEUX (2008): "Regression Discontinuity Designs: A Guide to Practice," *Journal of Econometrics*, 142, 615–635.

KATAGIRI, M., K. TAKAHASHI, AND J. SHINO (2022): "Bank of Japan's ETF purchase program and equity risk premium: a CAPM interpretation," .

KRISHNAMURTHY, A. AND A. VISSING-JORGENSEN (2011): "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," .

——— (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

KUBOTA, H. AND M. SHINTANI (2022a): "High-Frequency Identification of Monetary Policy Shocks in Japan," *The Japanese Economic Review*.

——— (2022b): "Macroeconomic Effects of Monetary Policy in Japan an Analysis Using Interest Rate Futures Surprises," SSRN Scholarly Paper 4015068, Social Science Research Network, Rochester, NY.

KUERSTEINER, G. M., D. C. PHILLIPS, AND M. VILLAMIZAR-VILLEGAS (2018): "Effective Sterilized Foreign Exchange Intervention? Evidence from a Rule-Based Policy," *Journal of International Economics*, 113, 118–138.

LI, J. J., N. D. PEARSON, AND Q. ZHANG (2021): "Impact of Demand Shocks on the Stock Market: Evidence from Chinese IPOs," SSRN Scholarly Paper 3731219, Social Science Research Network, Rochester, NY.

MCCRARY, J. (2008): "Manipulation of the Running Variable in the Regression Discontinuity Design: A Density Test," *Journal of Econometrics*, 142, 698–714.

NAKAMURA, E. AND J. STEINSSON (2018): "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect*," *The Quarterly Journal of Economics*, 133, 1283–1330.

PORTER, J. (2003): "Estimation in the Regression Discontinuity Model," *Unpublished Manuscript, Department of Economics, University of Wisconsin at Madison*, 2003, 5–19.

PORTER, J. AND P. YU (2015): "Regression Discontinuity Designs with Unknown Discontinuity Points: Testing and Estimation," *Journal of Econometrics*, 189, 132–147.

RAMBACHAN, A. AND N. SHEPHARD (2021): "When Do Common Time Series Estimands Have Nonparametric Causal Meaning?" .

ROMER, C. D. AND D. H. ROMER (2004): "A New Measure of Monetary Shocks: Derivation and Implications," *American Economic Review*, 94, 1055–1084.

SHIROTA, T. (2018): "Evaluating the Unconventional Monetary Policy in Stock Markets : A Semi-parametric Approach," 24.

VAYANOS, D. AND J.-L. VILA (2021): "A Preferred-Habitat Model of the Term Structure of Interest Rates," 100.

WALLACE, N. (1981): "A Modigliani-Miller Theorem for Open-Market Operations," *The American Economic Review*, 71, 267–274.