論　　文

Endogenous Fertility and Social Security*

By Masaya YASUOKA**

Abstract

Whether those of working age find employment depends on general economic conditions. Some will be unemployed, and it is impossible to forecast economic events perfectly. This uncertainty provides the motive for precautionary saving. Even if young households could afford to pay for childcare, they do not do so because of precautionary saving. Thus, policies to reduce precautionary saving are effective for raising expenditure on childcare. This paper develops an endogenous fertility model with uncertainty regarding future income in middle age and old age and examines how such uncertainty affects fertility. The results obtained in this study show that an increase in unemployment benefits can raise fertility in the model with uncertainty. No such result is derived in a model without uncertainty. In the model with uncertainty, income transfer policies have the same effect on fertility as child allowances. Moreover, this study shows that uncertainty with regard to labor income in old age also reduces fertility. Therefore, a decrease in uncertainty with regard to future income raises fertility.

JEL Classification Codes: J14, J13, J26
Keywords: Elderly labor, Fertility, Precautionary saving

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出生率内生化モデルと社会保障

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＜要旨＞

経済状況は就労世代の雇用に影響を与える。就労世代の一部は失業状態となる。私たちは将来の経済状況を完全に予測することはできない。不確実性は予備的貯蓄の動機をもたらす。たとえ、若年世代が子どもを育てる余裕があるとしても、予備的貯蓄のために子育てへの支出をしない。予備的貯蓄を減らす政策が子育て支出を増やすためには有効である。本稿は将来の不確実な所得を伴う出生率内生化モデルを設定し、壮年期と老年期における不確実性が出生率にどのような影響を与えるかを考察する。得られた結果は次の通りである。不確実性のあるモデル経済では失業給付の増加は出生率を引き上げる。しかしこの結果は不確実性のないモデル経済では得られない結果である。不確実性のあるモデルでは、出生率に与える影響の観点から所得移転政策が児童手当と同じ効果を持つ。さらに、本稿では高齢期における労働所得の不確実性は出生率を低水準に留めることを明らかにした。このことは、将来所得の不確実性の減少が出生率の増加をもたらすことを意味している。

JEL Classification Codes: J14, J13, J26

Keywords: 高齢者労働、出生率、予備的貯蓄

1. Introduction

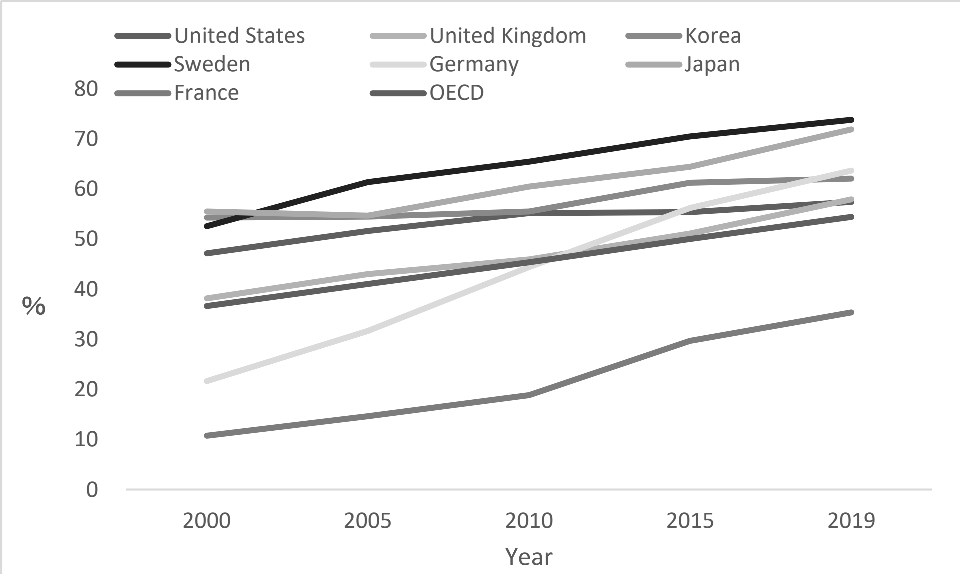
The aim of this paper is to examine how the uncertainty about the future affects fertility. Many studies in this field employ deterministic models and do not examine fertility decisions into a model with uncertainty. However, uncertainty plays an important role in households' allocation decisions.[1] Numerous studies examine the links between uncertainty and precautionary saving (Leland (1968), Caballero (1991), Liljas (1998), Picone et al. (1998), and Hemmi et al. (2007)). These papers find that uncertainty about the future reduces consumption in the present period and increases saving as individuals attempt to prepare for future eventualities. This, in short, is the motivation for precautionary saving. Thus, individuals engage in precautionary saving when they are young due to uncertainty about their situation when they will be older.

Real world examples of uncertainty about the future include pension reforms that reduce the level of pension benefits and uncertainty over wages in the future. Another source of uncertainty is the risk of unemployment. For instance, Murata (2003) empirically shows that anxiety about future income is a motive for precautionary saving. Meanwhile, Morikawa (2019) shows that uncertainty about how social security and tax reforms affect household income results in precautionary saving. In the theoretical literature, a number of studies examine the link between uncertainty and fertility. For example, Sugimoto and Nakagawa (2011) consider the irreversibility of childbirth and children's educational aptitude. Hirata (2012) considers the uncertainty of household income.[2] However, these studies do not consider the impact of policies on fertility in a model with uncertainty.

This paper considers income uncertainty. Specifically, the following sources of income uncertainty are considered. For those of working age, uncertainty derives from the risk of becoming unemployment, while for the elderly, both their work income and the level of their pension benefits are uncertain. Before COVID-19, unemployment in was very low.[3] However, unemployment rose as a result of COVID-19. However, not only those of working age face income uncertainty. The elderly also face income uncertainty because of the economic situation and the government's policies with regard to pensions. As shown in Figure 1, the labor participation rate of the elderly (those aged 60–64) in Japan is above the average of OECD countries and continues increasing. Therefore, it is important to consider the uncertainty of labor income in older age.

[1] For instance, an article in the Nikkei newspaper ("Korona fuan, shussho ritsu ni eikyo mo nichibei de ichiwari gen yosoku," in Japanese, August 21, 2020) highlights that uncertainty about future income can decrease fertility.
[2] Developing a Ramsey model with household income uncertainty, Hirata (2012) estimates the correlation between fertility and the variance of income. On the other hand, this paper considers the work income and social security of the elderly as concrete sources of household income uncertainty in an overlapping generations model to consider intergenerational transfers.
[3] The unemployment rate in Japan was 2.35 percent in 2019. However, in 2020, the unemployment rate rose to 2.77 percent, which was a smaller increase than in other countries. For instance, the unemployment rate in the United States increased from 3.67 percent in 2019 to 8.09 percent in 2020. (Data from OECD Data; online: https://data.oecd.org/)

Figure 1: Labor participation rate of 60–64 year olds (%)



Source: OECD Statistics; online: https://stats.oecd.org/.

Some studies have examined retirement and labor participation of the elderly. Matsuyama (2008) and Gong and Liu (2012) show that if retirement benefits are small, elderly people will continue working instead of retiring. Retirement benefits consist, for instance, of pension benefits. Maeda and Momota (2002) and Momota (2003) estimate the correlation between pension benefits and retirement. Conde-Ruiz and Galasso (2003, 2004) examine the impact of the pension system on the retirement age and show that retirement is correlated with wages for the elderly. Miyake and Yasuoka (2018) find that subsidies for elderly workers increase the labor supply and lead people to postpone retirement.

There exist many studies about endogenous fertility and policy. Van Groezen et al. (2003) show that child allowances can raise fertility. In a model on the quality and quantity of children, Zhang (1997) shows that child allowance raise fertility and reduce investment in children's education. In contrast, education subsidies raise investment in education and reduce fertility. Some papers examine the link between the labor supply of the elderly or life expectancy on the one hand and endogenous fertility on the other. For instance, Yakita (2001) shows that an increase in life expectancy reduces fertility. Meanwhile, Mizuno and Yakita (2013) find that working when older increases households' lifetime income and that fertility can be raised by an increase in expenditure on childcare.

The findings of the current paper can be summarized as follows. The risk of unemployment reduces fertility because of precautionary saving. However, an increase in unemployment benefits raises fertility. However, this effect is observed only in the presence of uncertainty. Moreover, examining the role of income uncertainty of the elderly shows that greater uncertainty reduced fertility. These findings suggest that, like child allowances, policies to reduce income uncertainty raise fertility.

The remainder of this paper is organized as follows. Section 2 develops a simple endogenous fertility model with uncertainty to show how uncertainty affects fertility. Section 3 examines how the probability of unemployment affects saving and fertility to consider the role of uncertainty. Section 4 presents an endogenous fertility model with elderly labor supply and no uncertainty to show how the saving and fertility are determined. Section 5 introduces uncertainty into the model presented in Section 4. Finally, Section 6 concludes this paper.

2. A simple endogenous fertility model with uncertainty

This section presents a simple endogenous fertility model with uncertainty. The following utility function is assumed:[4]

$$U_t = u(n_t) + E_t v(c_{t+1}), \tag{1}$$

where $u'(n_t) > 0, u''(n_t) < 0, v'(c_{t+1}) > 0, v''(c_{t+1}) < 0$. $u(n_t)$ and $v(c_{t+1})$ are defined as continuous functions. E_t denotes the expectations operator. The individual lives in two periods, labeled "young" and denoted by t and "old" denoted by $t+1$. The individual cares for children in the "young" period. n_t denotes the number of children. In the "old" period, the individual derives utility from consumption, c_{t+1}.

The budget constraint in the "young" period is given by

$$zn_t + s_t = w_t, \tag{2}$$

where z denotes the childcare cost per child. w_t denotes the wage rate, and the individual provides labor inelastically. s_t denotes saving for the consumption in the second, i.e., "old," period.

The budget constraint in the "old" period is given by

$$c_{t+1} = (1+r)s_t + w_{t+1}, \tag{3}$$

where r denotes the interest rate. w_{t+1} shows the wage rate in the second period. However, the wage rate in the second period is subject to uncertainty. There exist a "good state" and a "bad state" with regard to the wage rate in second period, with probability q and $1-q$, respectively ($0 < q < 1$). In the good state, w_{t+1} is given by w_{t+1}^h. On the other hand, in the bad state, w_{t+1} is given by w_{t+1}^l ($w_{t+1}^h > w_{t+1}^l$). Thus, the average wage rate $E_t w_{t+1}$ is given by

$$E_t w_{t+1} = q w_{t+1}^h + (1-q) w_{t+1}^l. \tag{4}$$

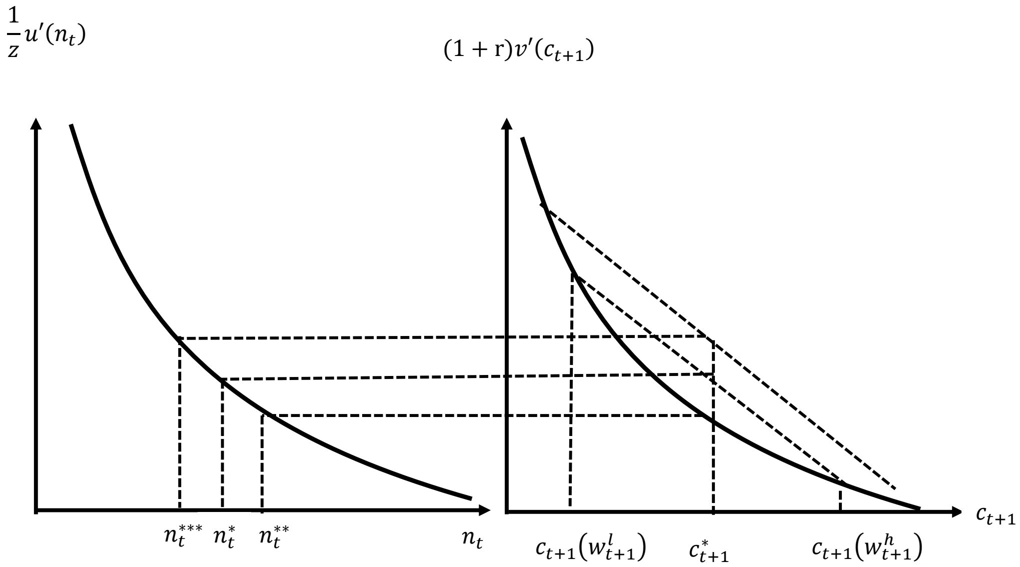
[4] The specific functional form is shown in Appendix A.

The optimal allocation to maximize utility (1) subject to budget constraints (2) and (3) is given by the following marginal condition:

$$\frac{1}{z}u'\left(\frac{w-s_t}{z}\right)=(1+r)E_t v'\big((1+r)s_t+w_{t+1}\big). \tag{5}$$

Optimal saving s_t^* is obtained to ensure that the optimality condition (5) holds. Then, substituting s_t^* into (2) and (3), fertility n_t^* and consumption c_{t+1}^* can be obtained as shown in Figure 2.

Figure 2: Decision of n_t



Without uncertainty, that is, if the individual knows the average level of wage rate in second period, the number of children is given by n_t^{**}, as shown in Figure 2. Without uncertainty, fertility is higher than in the model with uncertainty. In the model with uncertainty, the individual engages in precautionary saving. If the wage rate in second period falls into the "bad state," the individual's income in the "old" period is low. Therefore, to ensure an adequate level of consumption in the "old" period, the individual increases saving in the "young" period to be prepared for the "bad status." If the degree of uncertainty is higher, that is, the variance of wage rate is greater, the average marginal utility of c_{t+1} increases, as shown in Figure 2.[5] Therefore, an increase in the average marginal utility of consumption raises precautionary saving, leading to a decline in fertility, as shown by n_t^{***} in Figure 2.

3. Endogenous fertility with uncertainty and policies

This section presents an endogenous fertility model with uncertainty to examine how policies to

[5] Figure 2 shows one of the cases.

address income uncertainty affect fertility.

3.1 Unemployment model

This subsection examines the effects of income uncertainty by considering a model in which an individual's working life is divided into two phases: young adulthood and middle age. The following utility function is assumed:

$$U_t = \alpha \ln n_t + q(1-\alpha)\ln c_{t+1}^g + (1-q)(1-\alpha)\ln c_{t+1}^b, 0 < \alpha < 1. \tag{6}$$

n_t denotes the number of children, which is determined in young adulthood. c_{t+1}^g and c_{t+1}^b represent consumption in the good state and the bad state in middle age period, respectively. The budget constraints in young adulthood and middle age are as follows:

$$zn_t + s_t = w_t. \tag{7}$$

$$c_{t+1}^g = (1+r)s_t + (1-\tau)w_{t+1}. \tag{8}$$

$$c_{t+1}^b = (1+r)s_t + b_{t+1}. \tag{9}$$

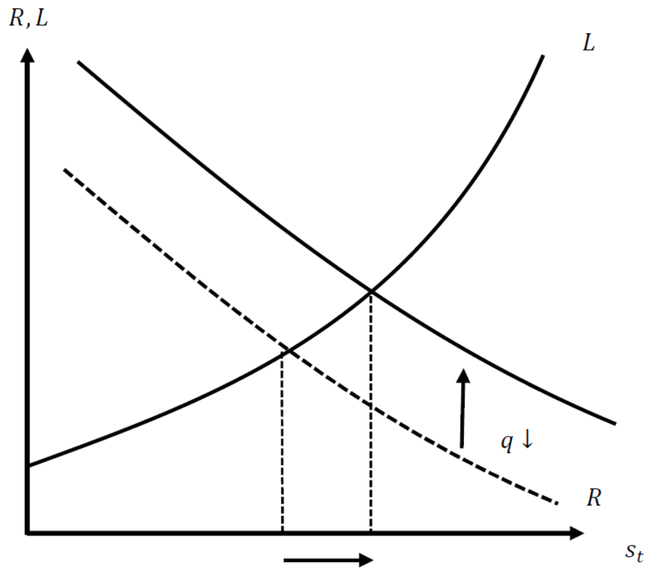
In young adulthood, individuals obtain a wage income. However, in middle age, some individuals obtain a wage income while others do not obtain a wage income because they are unemployed. $1-q$ $(0 < q < 1)$ denotes the probability of unemployment, and unemployment represents the bad state. On the other hand, employment represents the good state. The probability that an individual is employed is given by q. When an individual is unemployed, they received unemployment benefit b_{t+1}, which is financed through income tax τ.

Let us derive the intertemporal household allocations between young adulthood and middle age. In young adulthood, individuals face the possibility that they may be unemployed in middle age. Considering the maximization of the utility function (6) with the constraints (7) to (9), we obtain the optimal allocations of s_t. The optimal allocation of s_t needs to satisfy the following equation:

$$\frac{\alpha}{w_t - s_t} = \frac{q(1-\alpha)(1+r)}{(1+r)s_t + (1-\tau)w_{t+1}} + \frac{(1-q)(1-\alpha)(1+r)}{(1+r)s_t + b_{t+1}}. \tag{10}$$

Defining the left-hand and the right-hand sides of (10) as R and L, respectively, Figure 3 can be drawn and the optimal saving allocation s_t can be obtained. Given (7) to (9), the household's optimal allocations n_t, c_{t+1}^g and c_{t+1}^b can be obtained.

Figure 3: An increase in the probability of unemployment



The figure shows that, assuming that $(1-\tau)w_{t+1} > b_{t+1}$, an increase in the probability of unemployment raises saving.[6] Differentiating R with respect to q yields the following:

$$\frac{dR}{dq} = (1-\alpha)(1+r)\left(\frac{1}{(1+r)s_t + (1-\tau)w_{t+1}} - \frac{1}{(1+r)s_t + b_{t+1}}\right) < 0. \tag{11}$$

Therefore, an increase in the probability of unemployment $1-q$ shifts R upward and saving increases. This can be regarded as precautionary saving. Due to precautionary saving, fertility decreases.

Next, the effect of unemployment benefits on precautionary saving and fertility is examined. Unemployment benefits are financed through income taxes and the government budget constraint with regard to unemployment benefits is assumed to be the balanced budget constraint:

$$q\tau w_{t+1} = (1-q)b_{t+1} \rightarrow b_{t+1} = \frac{q\tau w_{t+1}}{1-q}. \tag{12}$$

Given (10) and (12) and differentiating R with respect to τ, this can be reduced as follows:

$$\frac{dR}{d\tau} = (1-\alpha)(1+r)qw_{t+1}\left(\frac{1}{\left((1+r)s_t + (1-\tau)w_{t+1}\right)^2} - \frac{1}{\left((1+r)s_t + \frac{q\tau w_{t+1}}{1-q}\right)^2}\right). \tag{13}$$

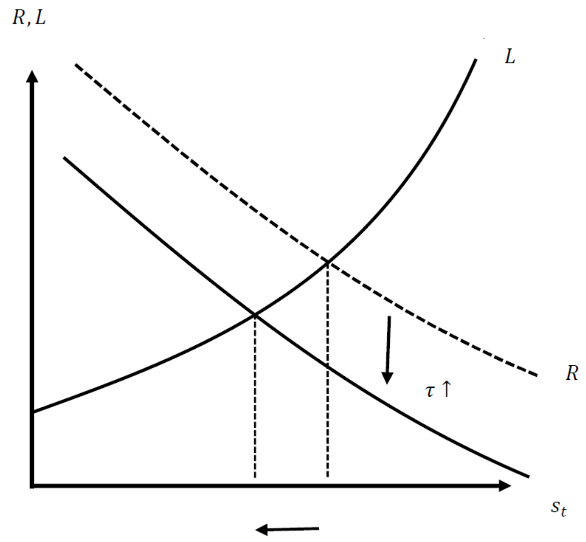
The condition for $\frac{dR}{d\tau} < 0$ to hold is given by

[6] This assumption is appropriate because $(1-\tau)w_{t+1} > b_{t+1}$ indicates that the disposable wage income is larger than the unemployment benefit.

$$\tau < 1 - q\,. \tag{14}$$

R shifts down, precautionary saving declines, and fertility increases as shown by Figure 4. Thus, the following proposition is obtained.

Figure 4: An increase in unemployment benefits



Proposition 1

Given $\tau < 1 - q$, an increase in unemployment benefits reduces saving and fertility increases.

Unemployment benefits represent an intra-generational income transfer. An increase in unemployment benefits reduces precautionary saving because income in the case of unemployment is higher. Child allowances and subsidies for childcare services are regarded as policies to increase fertility. However, the analysis here shows that, with uncertainty regarding future income, policies to reduce the income gap between being employed (=good state) and unemployed (=bad state) would also have the effect of raising fertility. Child allowances raise fertility because they reduce the net cost of having children. On the other hand, unemployment benefits raise fertility because of the income effect that households can afford to have children even when they become unemployed. While Fanti and Gori (2010) examine an endogenous fertility model with unemployment, they do not consider income uncertainty, so that the result obtained here is a new one because of income uncertainty.

3.2 Unemployment without uncertainty

Even if individuals can work in middle age and have sufficient income to raise children, income uncertainty reduces fertility. Unemployment benefits are regarded as insurance, allowing a certain income level. However, in a model with no uncertainty, there is no insurance effect. Therefore, the next step is check that without uncertainty in the model there is no effect on fertility. Without uncertainty, young adults will know their income when they are older. Thus, the number of children of individuals in employment in middle age, n_t^g , and unemployed in middle age, n_t^b are given by

$$n_t^g = \frac{\alpha}{z}\left(w_t + \frac{(1-\tau)w_{t+1}}{1+r}\right), \tag{15}$$

$$n_t^b = \frac{\alpha}{z}\left(w_t + \frac{b_{t+1}}{1+r}\right). \tag{16}$$

Thus, the average number of children, n_t^{ave}, is given by

$$n_t^{ave} = qn_t^g + (1-q)n_t^b. \tag{17}$$

An increase in unemployment benefit does not affect the number of children because

$$\frac{dn_t^{ave}}{d\tau} = 0. \tag{18}$$

In the model without uncertainty, the intragenerational income transfer in the form of unemployment benefits does not raise fertility. An increase in unemployment benefits raises the fertility of individuals that are unemployed in middle age. However, the number of children of individuals that are in employment in middle age will be lower because of the reduction in disposable household income as a result of the income transfer.

3.3 Labor force participation

Childcare can be divided into using childcare services and doing the childcare oneself. If individuals decide to do the childcare themselves, the amount of time available for work reduces by the time spent on childcare. If the childcare time per child is given by ϕ, the childcare time for n_t children is given by ϕn_t and labor time is given by $1-\phi n_t$. On the other hand, if individuals use childcare service, labor time is given by 1, since they spend no time on childcare.

In this subsection, it is assumed that the labor productivity of young adults, a, is uniformly distributed between [0, $\bar{a}$]. Then, if the cost of childcare services, z, is less than the opportunity cost of taking care of children, $\phi a w_t$, individuals use childcare services. Otherwise, they care for their children themselves. Thus, (7) changes to the following:

$$zn_t^c + s_t = aw_t, \tag{19}$$

$$s_t = (1-\phi n_t^n)aw_t. \tag{20}$$

n_t^c and n_t^n denotes the fertility of individuals that use and do not use childcare services, respectively. Given $0 < z < \phi w_t \bar{a}$, the following is obtained: $a^* = \frac{z}{\phi w_t}$. Thus, individuals with [0, a^*] do not use childcare services. On the other hand, individuals with [a^*, $\bar{a}$] do use childcare services. Average labor participation is given by the following:

$$\int_0^{a^*}(1-\phi n_t^n)\frac{1}{\bar{a}}da+\frac{\bar{a}-a^*}{\bar{a}}. \tag{21}$$

A decrease in childcare services cost z reduces a^*. As a result, the share of individuals working full-time increases and labor participation increases. Focusing on female labor participation, the model suggests that a decrease in the cost of childcare services raise female labor participation, which is consistent with some empirical results.

Let us consider uncertainty in middle age. If the probability of unemployment increases, precautionary saving increases and fertility decreases. For individuals that work full-time, labor time does not change. However, individuals that care for their children themselves, reduce the number of children, as shown in Figure 3, so that labor supply increases. Therefore, the risk of unemployment raises labor participation and reduces fertility. This result yields the following proposition.

Proposition 2

A decrease in q reduces fertility due to an increase in saving. As a result, labor participation rises because households with [0, a^*] reduce the number of children they have.

Even if a household consists not only of a woman but also a man, the results with regard female labor participation do not change. If we consider male labor income, the uncertainty of male labor income reduces fertility and increases female labor participation because of precautionary saving. However, since the total income of the couple is considered, the “bad” outcome of low income can be avoided. Even if the man is unemployed, the woman will have a job, and vice versa. Therefore, precautionary saving declines and then fertility increases.

This section did not consider income in old age. However, in practice, elderly people receive pension benefits and/or labor income. The following section therefore considers income in old age to develop a two-period decision model, consisting of young adulthood and middle age (“young age”) and old age (“old age”).

4. Model of labor supply by the elderly without uncertainty

This section considers the labor supply of the elderly. As shown in Figure 1, labor participation of the elderly has been increasing in OECD countries. The figure further shows that has Japan one of the highest labor participation rates of the elderly among OECD countries. A likely reason is the reform of the pension system in Japan, as part of which the pension age was raised by 5 years from 60 to 65 years of age. This means that individuals had to postpone their retirement, and for the elderly as a group, the share of work income in total income increased. Against this background, this section considers the role of labor market participation of the elderly in the context of the endogenous fertility model.

To start with, the model is considered without uncertainty for simplicity. The utility function is assumed to take the following form:

$$U_t = \alpha \ln n_t + (1-\alpha)\ln c_{t+1}, 0 < \alpha < 1. \tag{22}$$

The budget constraint in young age and old age respectively are as follows:

$$z n_t + s_t = w - T \tag{23}$$

$$c_{t+1} = (1+r)s_t + a w l_{t+1} + p_{t+1}(1 - l_{t+1}) \tag{24}$$

where l_{t+1} denotes the labor supply in old age. a denotes the individual labor productivity, which is uniformly distributed between $[0, \bar{a}]$ among the elderly. The wage rate w is constant over time. p_{t+1} denotes pension benefits, which are financed by lump-sum tax T. The decision of whether to supply labor or not is governed by the following inequality:

$$\begin{cases} aw \geq p_{t+1} \rightarrow l_{t+1} = 1 \\ aw < p_{t+1} \rightarrow l_{t+1} = 0 \end{cases} \tag{25}$$

The ability of working $a^* = \frac{p_{t+1}}{w}$ is given by $aw = p_{t+1}$. The optimal allocation to maximize utility (22) subject the budget constraints (23) and (24) can be derived. Fertility in the case where $l_{t+1} = 1$ and $l_{t+1} = 0$ are then respectively given by

$$n_t = \frac{\alpha\left(w - T + \frac{aw}{1+r}\right)}{z}, \ and \tag{26}$$

$$n_t = \frac{\alpha\left(w - T + \frac{p_{t+1}}{1+r}\right)}{z}. \tag{27}$$

If pension benefits are financed through a pay-as-you-go system, the government budget constraint for pensions is given by

$$p_{t+1}\int_0^{a^*} \frac{1}{\bar{a}} da = \bar{n}_t T \rightarrow p_{t+1} = \frac{\bar{a}\bar{n}_t T}{a^*}, \tag{28}$$

where $\bar{n}_t$ denotes the average fertility and $\frac{a^*}{\bar{a}}$ represent the share of pensioners in the elderly population. The average fertility rate is given by the following:[7]

$$\bar{n}_t = \frac{\alpha\left(w - T + \frac{\bar{a}w}{2(1+r)} + \frac{\bar{n}_t T}{2(1+r)}\right)}{z}. \tag{29}$$

A decrease in $\bar{a}$ leads to a decrease in the wage level of elderly workers, i.e., wage deflation,

[7] The average fertility rate is calculated as $\bar{n}_t = \int_{a^*}^{\bar{a}} \frac{\alpha\left(w-T+\frac{aw}{1+r}\right)}{z}\frac{1}{\bar{a}}da + \int_0^{a^*}\frac{\alpha\left(w-T+\frac{p_{t+1}}{1+r}\right)}{z}\frac{1}{\bar{a}}da$. The proof that a decrease in $\bar{a}$ reduces fertility is shown in Appendix B.

which has recently been observed in Japan. The effect of a decrease in $\bar{a}$ raises the share of pensioners $\frac{a^*}{\bar{a}}$. An increase in the $\frac{a^*}{\bar{a}}$ of pensioners reduces the level of pension benefits, p_{t+1}, since T is constant. A decrease in p_{t+1} raises labor participation of the elderly because of (25). A decrease in $\bar{a}$ decreases pension benefits, so that labor participation of the elderly increases. However, as shown by (29), fertility decreases. The model thus suggests that there is a negative link between labor participation of the elderly and fertility. However, studies such as Mizuno and Yakita (2013) suggest that there is a positive link between labor participation of the elderly and fertility. This paper arrives at the opposite results because of the income distribution and the pension system.

In Japan, the government provides subsidies for the employment of the elderly. Denoting the employment subsidy received by the elderly by ε, the budget constraint can be rewritten as follows

$$c_{t+1} = (1+r)s_t + (1+\varepsilon)awl_{t+1} - \bar{T} + p_{t+1}(1 - l_{t+1}), \tag{30}$$

where $\bar{T}$ denotes the lump-sum tax to pay for the subsidy for elderly workers. $\bar{T}$ is paid by the elderly workers. The government's budget constraint of the subsidy is given by $\varepsilon \int_{a^*}^{\bar{a}} \frac{aw}{\bar{a}} da = \bar{T}$. The threshold a^* is given by $(1+\varepsilon)a^* w = p_{t+1}$. An increase in ε raises the labor participation of the elderly as long as pension benefits are small.[8] Fertility in the case of a subsidy for labor participation of the elderly is given as follows:

$$\bar{n}_t = \frac{\alpha\left(w - T - \frac{\bar{T}}{1+r} + \frac{(1+\varepsilon)(\bar{a}^2 - a^{*2})w}{2\bar{a}(1+r)} + \frac{\bar{n}_t T}{1+r}\right)}{z}, \tag{31}$$

The subsidy for labor participation of the elderly leads to a positive link between labor participation of the elderly and fertility. This yields the following proposition.

Proposition 3

A decrease in the upper limit of wage income $\bar{a}$ leads to a negative link between labor participation of the elderly and fertility. The subsidy for employment of the elderly brings about a positive link.

The subsidy for employment of the elderly plays a similar role to the provision of a child allowance in that both raise fertility. However, while a child allowance raises fertility, it does not provide any advantages for the elderly. Therefore, from the perspective of intergenerational conflicts over social security, an employment subsidy for the elderly has the advantage that it raises fertility and avoids intergenerational conflict. Japan's current pension policies mean that if older people's work earnings are above a certain level, their pension benefits will be reduced. Without this reduction in benefits, labor supply of the elderly would increase. However, the policy actually adopted by the government is to provide subsidies for the employment of older people.

[8] See Appendix C for a detailed proof.

5. Model of labor supply by the elderly with uncertainty

This section considers the labor supply of the elderly in the model with endogenous fertility and uncertainty. Two cases of uncertainty are considered: uncertainty with regard to pension benefits and uncertainty with regard to the wage rate for the elderly.

5.1 Uncertainty with regard to pension benefits

To start with, we consider the case of uncertainty with regard to pension benefits. However, there is no uncertainty and distribution of elderly labor income aw. The utility function is assumed to be follows,

$$U_t = \alpha \ln n_t + (1-\alpha) E_t \ln c_{t+1}, 0 < \alpha < 1. \tag{32}$$

Moreover, the budget constraint in young age and old age respectively are as follows:

$$z n_t + s_t = w - T \tag{33}$$

$$c_{t+1} = (1+r)s_t + awl_{t+1} + p_{t+1}(1 - l_{t+1}). \tag{34}$$

Without uncertainty, as long as $aw < p_{t+1}$, the individual does not work to earn a wage income in old age. However, the uncertainty of pension benefit is assumed as follows,

$$p_{t+1} = \begin{cases} p+\sigma & \text{at probability } q \\ p-\sigma & \text{at probability } 1-q \end{cases} \tag{35}$$

Thus, the average level of pension benefits is given by $E_t p_{t+1} = q(p+\sigma) + (1-q)(p-\sigma)$.[9] It is further assumed that $p - \sigma < aw < E_t p_{t+1}$. That is, if households know that the level of pension benefits is $E_t p_{t+1}$ *ex ante*, they want to stop working to receive the pension benefit. Based on (32) to (35), the optimal level of saving is given by

$$\frac{\alpha}{w - T - s_t} = (1+r)(1-\alpha)\left(\frac{q}{(1+r)s_t + p + \sigma} + \frac{1-q}{(1+r)s_t + aw}\right). \tag{36}$$

With $p + \sigma > aw$, individuals select $l_{t+1} = 0$. They do not work and instead receive the pension benefit. On the other hand, with $p - \sigma < aw$, they select $l_{t+1} = 1$. They work to earn wage income and do not receive the pension benefit. If there is no uncertainty and they can receive the average level of pension benefits, the following equation (37) holds, and because $aw < E_t p_{t+1}$, they do not work and instead obtain the pension benefit, as shown by the following equation:

$$\frac{\alpha}{w - T - s_t} = \frac{(1+r)(1-\alpha)}{(1+r)s_t + E_t p_{t+1}}. \tag{37}$$

If households know they will obtain the average level of pension benefits, they do not consider

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[9] If the government runs the pension system successfully and it is fully funded, the pension benefit will be $p + \sigma$. Otherwise, the pension benefit will be given by $p - \sigma$.

working in old age. However, in the model with uncertainty, they take the possibility of a bad pension benefit state into account and will want to work in old age to compensate for the loss in pension benefits in the bad state. In this model, there are two ways in which individuals can deal with uncertainty: one is to engage in precautionary saving, and the other is to work in old age. In young age, households consider precautionary saving and the possibility of working in old age if the pension benefit is low. Compared with the model in which the elderly cannot work, precautionary saving is lower in the model in which the elderly can work, because households can deal with the uncertainty of a low income in old age with not only by saving but also by working in old age.

5.2 Uncertainty with regard to the wage rate for the elderly

Second, uncertainty with regard to the wage rate for elderly is considered. For simplicity, it is assumed there are no pension benefits. However, even if pension benefits were included, the results obtained in this subsection would remain largely unchange. As in Section 4, it is assumed ability a is uniformly distributed across elderly people at $[0, \bar{a}]$. Further, it is assumed that while households do not know their ability a in old age when they are still young, they do know their ability once they are old. Optimal savings under these assumptions can be derived as follows:

$$\frac{\alpha}{w - s_t} = (1+r)(1-\alpha)\int_0^{\bar{a}} \frac{1}{(1+r)s_t + aw}\frac{1}{\bar{a}}da, \tag{38}$$

that is,

$$\frac{\alpha}{w - s_t} = \frac{(1+r)(1-\alpha)}{w}\ln\frac{(1+r)s_t + \bar{a}w}{(1+r)s_t}\frac{1}{\bar{a}}. \tag{39}$$

The left-hand-side of (39) is an increasing function of s_t, while the right-hand-side of is a decreasing function of s_t. We can obtain the intersect as the interior solution of s_t. In the model with uncertainty, every young individual has a precautionary saving motive. However, without uncertainty, individuals that know that their ability in old age will be high save less because they know that they will have a high income in old age and they therefore do not need to save. Precautionary saving reduces fertility. Even if elderly people have a large income, they cannot increase the fertility because of the problem of timing of childcare. It is important to reduce the precautionary saving to deal with fewer children.[10]

In this study, children are regarded as a consumption good that raises utility. However, children could also be regarded as an investment good, where older people receive gifts (income transfers) from their children. In this case, if the income transfers are not uncertain, fertility will be higher, since

[10] If we consider the case the of retirement, where individuals receive pension benefits once they have retired, it is possible to develop a model with retirement and pension benefits. Defining $\bar{l}$ as the labor supply time in old age, 1-$\bar{l}$ denotes the time that the elderly people can obtain the pension benefit. The budget constraint of the pay-as-you-go pension is then given by $\bar{n}_t T = (1-\bar{l})p_{t+1}$. Thus, (24) changes to $c_{t+1} = (1+r)s_t + aw\bar{l} + p_{t+1}(1-\bar{l})$ and saving is given by $\frac{\alpha}{w-T-s_t} = \frac{(1+r)(1-\alpha)}{w\bar{l}}\ln\frac{(1+r)s_t+(1-\bar{l})p_{t+1}+\bar{a}w\bar{l}}{(1+r)s_t+(1-\bar{l})p_{t+1}}\frac{1}{\bar{a}}$. Therefore, an increase in $\bar{l}$ reduces saving s_t if the decrease in $\frac{(1+r)(1-\alpha)}{w\bar{l}}$ is larger than the effect of the increase in $\ln\frac{(1+r)s_t+(1-\bar{l})p_{t+1}+\bar{a}w\bar{l}}{(1+r)s_t+(1-\bar{l})p_{t+1}}\frac{1}{\bar{a}}$.

in the model with uncertainty households would want a certain income. However, as long as the income transfers are uncertain, the results obtained in this study remain unchanged even if children are regarded as an investment.

6. Concluding remarks

This study examined how uncertainty affects the household decision of the number of children. Uncertainty about future income decreases fertility because of precautionary saving and decreases expenditure on childcare. Therefore, fertility could be increased if people have less reason for precautionary saving. One possible policy to reduce the need for precautionary saving would be an increase in unemployment benefits. In the model without uncertainty regarding employment, however, fertility does not increase even if unemployment benefits are provided.

Labor supply of the elderly is linked to pension policy. If labor income of the elderly is high, elderly people continue to work in old age. However, because of uncertainty for elderly labor supply, the younger people consider low elderly labor income and decide the demand for the precautionary saving. Even if individuals have a lifetime income and older people know *ex post* that they have a high lifetime income, fertility does not increase because fertility is determined in young age.

Social security for older people such as pensions affect fertility. Therefore, not only child allowances but also social security for older people should be considered as a way to tackle a low birth rate. Even if child allowances are provided to raise fertility, policies to reduce income uncertainty, such as unemployment benefits, are effective in increasing fertility in the model with uncertainty.

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Appendix

Appendix A. Specific form of the utility function

Two alternative utility functions are considered to incorporate endogenous fertility. The first is the following quasi-linear utility function with regard to consumption:

$$U_t = \alpha \ln n_t + (1-\alpha)\left(c_{t+1} - \frac{c_{t+1}^2}{2}\right) \quad \text{(A.1)}$$

Thus, the marginal utility of c_{t+1} can be derived as $1 - c_{t+1}$. Then, the curve in the right of Figure 2 changes to the linear. Therefore, the individual considers the average wage rate $E_t w_{t+1}$ to maximize utility. The utility maximization condition for (A.1) is given by

$$\frac{\alpha}{w_t - s_t} = (1-\alpha)(1+r)(1-(1+r)s_t - E_t w_{t+1}) \quad \text{(A.2)}$$

The left-hand side of (A.2) is an increasing function of s_t. On the other hand, the right-hand side of (A.2) is a decreasing function of s_t. Therefore, as long as $\frac{\alpha}{w_t} < (1-\alpha)(1+r)\left(1 - \frac{E_t w_{t+1}}{1+r}\right)$, the interior solution of s_t can be obtained. Otherwise, no consumption allocation can be derived. A mean-preserving increase in variance leaves saving unchanged, since (A.3) does not include the variance of wage rate and depends only on the average $E_t w_{t+1}$. That is, individuals have no motive for precautionary saving. Therefore, even if there is uncertainty with regard to wage rate, as long as the marginal utility of consumption is linear, individuals consider the average wage rate level and no variance to maximize utility.

Next, let us consider the second utility function. Sometimes, a constant relative risk aversion (CRRA) given by $v(c_{t+1}) = \frac{c_{t+1}^{1-\gamma}}{1-\gamma}, 0 \leq \gamma$ is assumed. When $\gamma = 1$, the logarithm utility function is given by $v(c_{t+1}) = \ln c_{t+1}$. When $\gamma = 0$, this yields perfect substitution, $v(c_{t+1}) = c_{t+1}$. Thus, utility function (1) takes the following form:

$$U_t = u(n_t) + E_t c_{t+1}. \quad \text{(A.3)}$$

Based on the budget constraint (2) and (3), using $\frac{dU_t}{ds_t} = 0$, the optimal allocation can then be derived follows:

$$u'(n_t) = z(1+r). \quad \text{(A.4)}$$

(A.4) shows that fertility n_t depends not on the variance of the wage rate in the second period but on $z(1+r)$. As shown by these two utility functions, if the marginal utility of consumption is linear or constant, fertility does not depend on the variance of future income, that is, the uncertainty of future income.

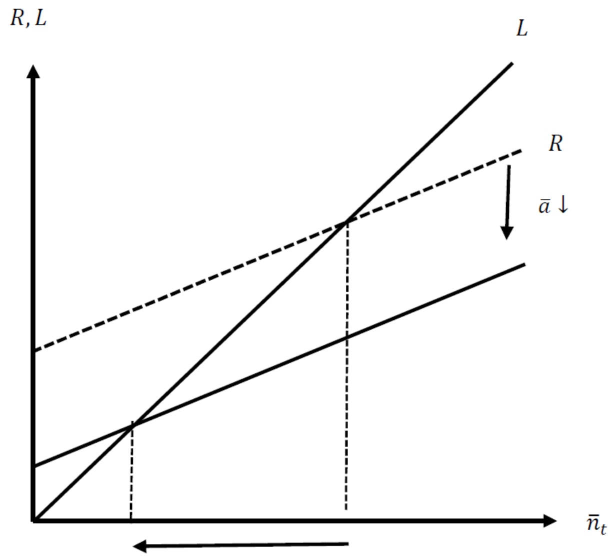
Appendix B. Proof of $\frac{\bar{n}_t}{\bar{a}} > 0$

(25)-(27) yield the following average fertility:

$$\bar{n}_t = \frac{\alpha}{z}\left(w - T + \frac{\left(\bar{a}^2 - \frac{\bar{a}\bar{n}_t T}{w}\right)w}{2\bar{a}(1+r)} + \frac{\bar{n}_t T}{1+r}\right) = \frac{\alpha}{z}\left(w - T + \frac{\bar{a}w}{2(1+r)} + \frac{\bar{n}_t T}{2(1+r)}\right). \qquad \text{(B.1)}$$

Defining the left- and the right-hand sides of (B.1) as L and R, respectively, Figure 5 can be drawn, where the intersect shows the average fertility. A decrease in $\bar{a}$ shifts R down and the intersect moves to left, representing a decrease in average fertility.

Figure 5: A Decrease in $\bar{a}$ and Average Fertility



Appendix C. Effect of a subsidy for elderly workers

Given $\varepsilon \int_{a^*}^{\bar{a}} \frac{aw}{\bar{a}} da = \bar{T}$, $(1+\varepsilon)a^* w = p_{t+1}$ and (31) and differentiating $\bar{n}_t$ with respect to ε at the approximation of $\varepsilon = 0$ yields the following:

$$\frac{d\bar{n}_t}{d\varepsilon} = \frac{\alpha w a^{*2}}{2\bar{a}z(1+r)\left(1 - \frac{\alpha T}{2z(1+r)}\right)} > 0. \qquad \text{(C.1)}$$

Because of the interior solution of $\bar{n}_t$, $1 - \frac{\alpha T}{2z(1+r)} > 0$.

$\frac{da^*}{d\varepsilon}$ is given by the following

$$\frac{da^*}{d\varepsilon} = \frac{1}{2wa^*}\left(\bar{a}T\frac{d\bar{n}_t}{d\varepsilon} - wa^{*2}\right) = \frac{a^*}{2}\left(\frac{\alpha T}{2z(1+r)}\frac{1}{1 - \frac{\alpha T}{2z(1+r)}} - 1\right). \qquad \text{(C.2)}$$

Defining T^* to satisfy $\frac{1}{\frac{2z(1+r)}{\alpha T} - 1} = 1$, the subsidy for elderly workers raises labor participation of the elderly as long as $T < T^*$.