An introduction to DYNARE

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The views expressed herein are ours and do not necessarily represent the views of Banque de France.

Outline

1. Introduction
2. A New Keynesian simple model
3. Writing a model in Dynare
4. Solving and simulating
5. Estimating a DSGE model with Dynare

DSGE models

- Microfoundations: agents optimize an objective function;
- Agents are forward looking: intertemporal optimization;
- Rational expectation hypothesis;
- General equilibrium: markets interact;
- Stochastic shocks push economy away from equilibrium;
- Endogenous dynamics bring the model back to equilibrium;
- We use numerical methods to approximate the solution of such models.

1. computes the solution of deterministic models (arbitrary accuracy),
2. computes first and second order approximation to solution of stochastic models,
3. estimates (maximum likelihood or Bayesian approach) parameters of DSGE models, for linear and non-linear models.
4. check for identification of estimated parameters
5. computes optimal policy
6. performs global sensitivity analysis of a model,
7. estimates BVAR and Markov-Switching Bayesian VAR models.

A simple Neo-Keynesian model

- A simple model from [? ?]. Similar to [?] or [?].
- Main differences with RBC models:
  1. imperfect competition
  2. nominal rigidities
- Consequence: monetary policy matters

The representative household maximizes

$$\mathbb{E}_0 \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\left(C_{t+s}/A_{t+s}\right)^{1-\tau} - 1}{1-\tau} + \chi_M \ln\left(\frac{M_{t+s}}{P_{t+s}}\right) - \chi_H H_{t+s} \right\},$$

with:

- C_t: consumption index, in period t
- A_t: evel of technology
- $\frac{M_t}{P_t}$: real money balances,
- H_t: hours worked,
- $\frac{1}{\tau}$: elasticity of intertemporal substitution,
- χ_M: scale factor
- χ_H: scale factor.

Household's budget constraint

$$P_t C_t + B_t + M_t - M_{t-} \; + T_t = P_t W_t H_t + R_{t-} \; B_{t-} \; + P_t D_t,$$

with

P_t: price index for final goods,

W_t: real wage,

B_t: nominal government bond,

T_t: lump-sum taxes,

D_t: dividends received from monopolistic firms.

In addition, the usual transversality condition applies, excluding Ponzi schemes.

Final good sector firms

The representative firm producing the final good assembles a continuum of intermediate goods indexed by $j \in [0,1]$ in a perfectly competitive manner:

$$Y_t = \int^{\infty} Y_t(j)^{-\nu} dj^{\frac{1}{1-\nu}},$$

with:

Y_t: final good production index,

$Y_t(j)$: good j production,

$\bar{\nu}$: demand elasticity.

Final good price index

Under optimal behaviour, the price of the final good is

$$P_t = \left(\int^{\infty} P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}},$$

with

P_t: price index of final good,

$P_t(j)$: price index of good j.

- Intermediate good j is produced by a monopolist with technology

$$Y_{(}j) = A_t N_t(j),$$

where

$N_t(j)$: labor input of firm j.

- Firms face a uadratic adjustment cost when they change their price:

$$AC(j) = \frac{\phi}{2} \quad \frac{P_t(j)}{P_{t-}\ (j)} - \pi \quad {}^{2}$$

with

π: steady state inflation rate

ϕ: adjustment cost parameter

Intermediate goods firms (II)

Intermediate goods firms maximize the present value of future profits:

$$\mathbb{E}_t \quad \sum_{s=}^{\infty} \beta^s Q_{t+s} \quad \frac{P_{t+s}(j)}{P_{t+s}} \quad Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - AC_{t+s}(j) \quad ,$$

with

Q_{t+s}: marginal value of a unit of consumption good in period $t+s$

Monetary policy

The central bank stabilizes inflation and its policy is represented by an interest rate feedback rule:

$$R_t = R_{t-}^{\rho_R} \left(r\pi^\star \left(\frac{\pi_t}{\pi^\star}\right)^{\psi_1} \frac{Y_t}{Y_t^\star}^{\psi_2} \right)^{-\rho_R} e^{\epsilon_{R,t}},$$

with

- r: steady state real interest rate,
- π_t: gross inflation rate (P_t/P_{t-})
- $\pi^\star$: target inflation rate,
- $Y_t^\star$: potential output,
- $\epsilon_{R,t}$ monetary policy shock.

Fiscal policy

The government consumes a fraction ζ_t of output:

$$G_t = \zeta_t Y_t,$$

with

G_t: government expenditures.

The government levies a lump-sum tax T_t in order to balance the budget. The government budget constraint is

$$P_t G_t + R_{t-} \ B_{t-} \ = T_t + B_t + M_t - M_{t-} \ .$$

Market clearing

The following equilibrium relationships prevail:

$$
\begin{aligned}
Y_t &= C_t + G_t + AC_t \\
H_t &= N_t
\end{aligned}
$$

Exogenous processes (I)

There are three shocks in the model:

1. aggregate productivity A_t
2. share of government expenditures in total output $zeta_t$
3. monetary policy shock, $\epsilon_{R,t}$

$\epsilon_{R,t}$ is serially uncorrelated.

Exogneous preocesses (II)

A_t evolves according to

$$\ln A_t = \ln \gamma + \ln A_{t-} \; + \ln z_t,$$

and

$$\ln z_t = \rho_z \ln z_{t-} \; + \epsilon_{z,t},$$

with

γ: long run rate of growth of technology.

Define

$$g_t = \frac{1}{1-\zeta_t}$$

and

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-} \ + \epsilon_{g,t},$$

with

g: steady state share of government expenditures.

Potential output

Potential output is defined as the output that would prevail in absence of nominal rigidities:

$$Y_t^\star = (1-\nu)^{\frac{1}{\tau}} A_t g_t.$$

$$1 = \beta \mathbb{E}_t \quad \frac{C_{t+}}{A_{t+}} \frac{A_t}{C_t} \quad ^{-\tau} \frac{A_t}{A_{t+}} \frac{R_t}{\pi_t} \quad ,$$

$$1 = \frac{1}{\nu}\left[1 - \quad \frac{C_t}{A_t} \quad ^{\tau}\right] + \phi\left(\pi_t - \pi\right)\left[\left(1 - \frac{1}{2\nu} \quad \pi_t + \frac{\pi}{2\nu}\right]\right.$$

$$- \phi\beta\mathbb{E}_t \quad \frac{C_{t+}}{A_{t+}} \frac{A_t}{C_t} \quad ^{-\tau} \frac{Y_{t+}}{A_{t+}} \frac{Y_t}{A_t} \left(\pi_{t+} \quad - \pi\right) \pi_{t+} \quad ,$$

$$Y_t^\star = (1-\nu)^{\frac{1}{\tau}} A_t g_t,$$

$$R_t = R_{t-}^{\rho_R} \quad r\pi^\star \quad \frac{\pi_t}{\pi^\star} \quad ^{\psi_1} \quad \frac{Y_t}{Y_t^\star} \quad ^{\psi_2} \quad ^{-\rho_R} \quad e^{\epsilon_{R,t}},$$

$$\text{n } A_t = \ln \gamma + \ln A_{t-} \quad + z_t,$$

$$\ln z_t = \rho_z \ln z_{t-} \quad + \epsilon_{z,t},$$

$$\ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-} \quad + \epsilon_{g,t},$$

Detrending the model

- define detrended variables

$$c_t = \frac{C_t}{A_t}$$
$$y_t = \frac{Y_t}{A_t}$$

- steady state:

$$c = (1-\nu)^{\frac{1}{\tau}}$$
$$y = g(1-\nu)^{\frac{1}{\tau}}$$
$$r = \frac{\gamma}{\beta}R = r\pi^{\star}$$

- define $\hat{x}_t = \ln\left(\hat{x}_t/x\right)$ as the percentage deviation of variable x_t.

$$1 = \mathbb{E}_t \left\{ e^{-\tau \hat{c}_{t+1} + \tau \hat{c}_t + \hat{R}_t - \hat{z}_{t+1} - \hat{\pi}_{t+1}} \right\},$$

$$\frac{1-\nu}{\nu \phi \pi^2} \quad e^{\tau \hat{c}_t} - 1 \quad = \quad e^{\hat{\pi}_t} - 1 \qquad 1 - \frac{1}{2\nu} \quad e^{\hat{\pi}_t} + \frac{1}{2\nu}$$

$$- \beta \mathbb{E}_t \left\{ \left(e^{\hat{\pi}_{t+1}} - 1 \quad e^{-\tau \hat{c}_{t+1} + \tau \hat{c}_t + \hat{y}_{t+1} \hat{y}_t + \hat{\pi}_{t+1}} \right\}$$

$$e^{\hat{c}_t - \hat{y}_t} = e^{-g_t} - \frac{\phi \pi^2 g}{2} \quad e^{\hat{\pi}_t} - 1 \quad ^2$$

$$\hat{R}_t = \rho_R \hat{R}_{t-} \quad + (1-\rho_R)\left(\psi \ \hat{\pi}_t + \psi_2 \left(\hat{y}_t - \hat{g}_t\right)\right) + \epsilon_{R,t}$$

$$\hat{z}_t = \rho_z \hat{z}_{t-} \quad + \epsilon_{z,t},$$

$$\hat{g}_t = \rho_g \hat{g}_{t-} \quad + \epsilon_{g,t},$$

Measurement equations

$$
\begin{aligned}
YGR_t &= \gamma^{(Q)} + 100\left(\hat{y}_t - \hat{y}_{t-} \;\; + z_t\right) \\
INFL_t &= \pi^{(A)} + 400\hat{\pi}_t \\
INT_t &= \pi^{(A)} + r^{(A)} + 4\gamma^{(Q)} + 400\hat{R}_t
\end{aligned}
$$

with

$\gamma^{(Q)}$: rate of growth of output in percentage
$\pi^{(A)}$: annual inflation rate in percentage
$r^{(A)}$: annualized interest rate in percentage

Note that

$$
\begin{aligned}
\gamma &= 1 + \frac{\gamma^{(Q)}}{100} \\
\beta &= \frac{1}{1 + r^{(A)}/400} \\
\pi &= 1 + \frac{\pi^{(A)}}{400}
\end{aligned}
$$

Dynare model file (I)

```
var c R z pi y YGR INFL INT;
varexo e_R e_g e_z;

parameters tau nu gbar rho_R rho_g rho_z
           gamma_Q pi_A r_A kappa psi_1 psi_2;
```

Dynare model file (II)

```
tau =    2;
nu =     0.1;
kappa =  0.3;
beta =   0.99;
gbar =   1/0.85;
rho_R =  0.5;
rho_g =  0.8;
rho_z =  0.66;
pi_A =   4.0;
r_A =    0.8;
gamma_Q = 0.4;
psi_1 = 1.5;
psi_2 = 0.5;
```

Dynare model file (III)

```
model;
# pibar =  1 + pi_A/400;
# phi = tau*(1 - nu)/(nu*pibar^2*kappa);
# gamma = 1 + gamma_Q/100;
# beta = 1/(1+r_A/400);
```

Dynare model file (IV)

```
exp(-tau*c(+1) + tau*c + R - z(+1) - pi(+1)) = 1;
((1-nu)/(nu*phi*pibar^2))*(exp(tau*c) - 1) =
  (exp(pi) - 1)*((1 - 1/(2*nu))*exp(pi) + 1/(2*nu))
  - beta*(exp(pi(+1)) - 1)*exp(-tau*c(+1) + tau*c
  + y(+1) - y + pi(+1));
exp(c - y) = exp(-g)
           - (phi*pibar^2*gbar/2)*(exp(pi) - 1)^2;
R = rho_R*R(-1) + (1-rho_R)*(psi_1*pi + psi_2*(y - g))
    + e_R;
g = rho_g*g(-1) + e_g;
z = rho_z*z(-1) + e_z;
YGR = gamma_Q + 100*(y - y(-1) + z);
INFL = pi_A + 400*pi;
INT = pi_A + r_A + 4*gamma_Q + 400*R;
end;
```

Dynare model file (V)

```
steady_state_model;
c = 0;
R = 0;
z = 0;
pi = 0;
y = 0;
g = 0;
YGR = gamma_Q;
INFL = pi_A;
INT = pi_A + r_A + 4*gamma_Q;
end;

steady;
```

Dynare model file (VI)

```
shocks;
var e_R; stderr  0.003;
var e_g; stderr  0.004;
var e_z; stderr  0.004;
end;

stoch_simul(order=1);
```

Dynare output (I)

```
STEADY-STATE RESULTS:

c         0
R         0
z         0
pi        0
y         0
g         0
dgdp      0.4
pi_obs    4
R_obs     6.4
```

Dynare output (II)

```
MODEL SUMMARY

  Number of variables:          9
  Number of stochastic shocks:  3
  Number of state variables:    4
  Number of jumpers:            4
  Number of static variables:   3


MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS

Variables        e_R        e_g        e_z
e_R         0.000009   0.000000   0.000000
e_g         0.000000   0.000016   0.000000
e_z         0.000000   0.000000   0.000016
```

Dynare output (III)

```
POLICY AND TRANSITION FUNCTIONS
                                  c              R
Constant                          0              0
R(-1)                     -0.282604       0.333974
g(-1)                             0              0
z(-1)                      0.294811       0.209595
y(-1)                             0              0
e_R                       -0.565209       0.667947
e_g                               0              0
e_z                        0.446684       0.317568
```

Approximated decision rule

$$\hat{c}_t = -0.283\hat{R}_{t-} \quad + 0.295]\hat{z}_{t-} \quad - 0.565\epsilon_{R,t} + 0.447\epsilon_{z,t}$$

Dynare output (IV)

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
c	0.0000	0.0027	0.0000
R	0.0000	0.0031	0.0000
z	0.0000	0.0053	0.0000
pi	0.0000	0.0015	0.0000
y	0.0000	0.0072	0.0001
g	0.0000	0.0067	0.0000
dgdp	0.4000	0.7869	0.6192
pi_obs	4.0000	0.6120	0.3746
R_obs	6.4000	1.2366	1.5291

Dynare output (V)

VARIANCE DECOMPOSITION (in percent)

	e_R	e_g	e_z
c	43.91	0.00	56.09
R	47.29	0.00	52.71
z	0.00	0.00	100.00
pi	27.99	0.00	72.01
y	6.25	85.78	7.98
g	0.00	100.00	0.00
dgdp	6.96	28.71	64.33
pi_obs	27.99	0.00	72.01
R_obs	47.29	0.00	52.71

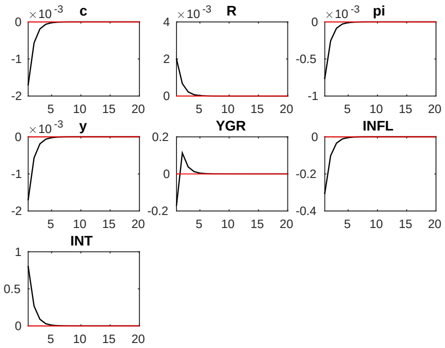
```
MATRIX OF CORRELATIONS

Variables       c       R       z       pi
c          1.0000  0.0208  0.7270  0.9844
R          0.0208  1.0000  0.7016  0.1961
z          0.7270  0.7016  1.0000  0.8363
pi         0.9844  0.1961  0.8363  1.0000
y          0.3772  0.0078  0.2742  0.3713
g          0.0000  0.0000  0.0000  0.0000
dgdp       0.6942  0.3890  0.7619  0.7493
pi_obs     0.9844  0.1961  0.8363  1.0000
R_obs      0.0208  1.0000  0.7016  0.1961
```

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
c	0.4137	0.1907	0.0984	0.0558	0.0337
R	0.5872	0.3633	0.2316	0.1502	0.0982
z	0.6600	0.4356	0.2875	0.1897	0.1252
pi	0.4769	0.2536	0.1470	0.0902	0.0572
y	0.7450	0.5761	0.4532	0.3593	0.2859
g	0.8000	0.6400	0.5120	0.4096	0.3277
dgdp	0.1808	0.1482	0.1055	0.0707	0.0459
pi_obs	0.4769	0.2536	0.1470	0.0902	0.0572
R_obs	0.5872	0.3633	0.2316	0.1502	0.0982

Impulse response function to monetary policy shock



ESTIMATION

Calibration versus estimation

Calibration:

- Models represents only some aspects of reality.
- Finding calibration that reproduces these aspects.
- Shortcoming: non indication of calibration uncertainty

Bayesian estimation:

- Combining previous information (previous studies, microeconomic studies) with data.
- The posterior distribution displays uncertainty surrounding parameter estimates.
- Bayesian approach throw a bridge between calibration and classical methods.

Ba esian paradigm (motivations)

- Experience shows that it is quite difficult to estimate a DSGE model by maximum likelihood.
  1. Data are not informative enough... The likelihood is flat in some directions (identification issue). This suggests that (when possible) we should use other sources of information.
  2. DSGE models are misspecified. When a DSGE is estimated by ML or with a "non informative" Bayesian approach (uniform priors) the estimated parameters are often found to be incredible. Using prior informations we can shrink the estimates towards sensible values.
- A related motivation is the relative lack of precision of ML. Prior information reduces the uncertainty.
- Final y the Bayesian approach allows easier comparison of (non nested) models.

Ba esian estimation main steps

1. priors specification as probability distributions,
2. computation of posterior distribution, on the basis of prior distribution and likelihood,
   - computing posterior mode,
   - simulating posterior distribution,
3. computing data marginal density useful for models comparison,
4. computing posterior predictive density
5. computing posterior distribution of IRFs, forecasts, etc . . .

Estimation of DSGE models

- only some variables are observed,
- statistical model: unobserved components model,
- likelihood must be evaluated through a state space representation of the model and the Kalman filter,
- there must be at least as many shocks as observed variables,
- otherwise, the model suffers from stochastic singularity.

Choosing priors

- Domain of definition:

Normal	$\mathbb{R}$
Uniform(a,b)	$[a, b]$
Gamma	$\mathbb{R}^+$
Beta	$[0, 1]$
Inverted gamma	$\mathbb{R}^+$

- mean prior: expected value,
- a small standard deviation means a tight prior,
- in Dynare, priors are orthogonal,
- implicit prior: unicity of stable equilibrium.

Dynare Estimation (I)

```
var c R z pi y g YGR INFL INT;
varexo e_R e_g e_z;

parameters tau nu gbar rho_R rho_g rho_z gamma_Q pi_A
           r_A kappa psi_1 psi_2;

nu =      0.1;
beta =    0.99;
gbar =    1/0.85;
```

Dynare Estimation (II)

```
model;
# pibar =  1 + pi_A/400;
# phi = tau*(1 - nu)/(nu*pibar^2*kappa);
# gamma = 1 + gamma_Q/100;
# beta = 1/(1+r_A/400);
```

Dynare Estimation (III)

```
exp(-tau*c(+1) + tau*c + R - z(+1) - pi(+1)) = 1;
((1-nu)/(nu*phi*pibar^2))*(exp(tau*c) - 1) =
  (exp(pi) - 1)*((1 - 1/(2*nu))*exp(pi) + 1/(2*nu))
  - beta*(exp(pi(+1)) - 1)*exp(-tau*c(+1) + tau*c
  + y(+1) - y + pi(+1));
exp(c - y) = exp(-g) - (phi*pibar^2*gbar/2)
                               *(exp(pi) - 1)^2;
R = rho_R*R(-1) + (1-rho_R)*(psi_1*pi + psi_2*(y - g))
    + e_R/100;
g = rho_g*g(-1) + e_g/100;
z = rho_z*z(-1) + e_z/100;
YGR = gamma_Q + 100*(y - y(-1) + z);
INFL = pi_A + 400*pi;
INT = pi_A + r_A + 4*gamma_Q + 400*R;
end;
```

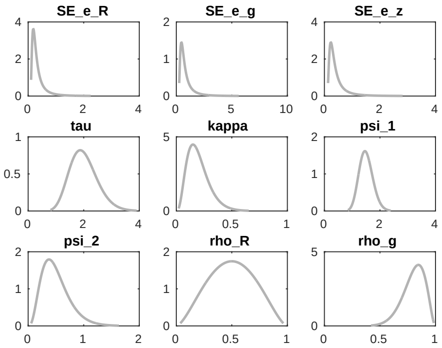
Dynare Estimation (IV)

```
steady_state_model;
c = 0;
R = 0;
z = 0;
pi = 0;
y = 0;
g = 0;
YGR = gamma_Q;
INFL = pi_A;
INT = pi_A + r_A + 4*gamma_Q;
end;
```

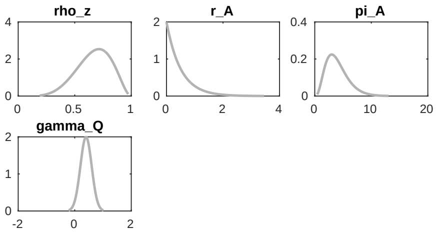
Dynare Estimation (V)

```
estimated_params;
tau, gamma_pdf, 2.0, 0.5;
kappa, gamma_pdf, 0.2, 0.1;
psi_1, gamma_pdf, 1.5, 0.25;
psi_2, gamma_pdf, 0.5, 0.25;
rho_R, beta_pdf, 0.5, 0.2;
rho_g, beta_pdf, 0.8, 0.1;
rho_z, beta_pdf, 0.666, 0.15;
r_A, gamma_pdf, 0.5, 0.5;
pi_A, gamma_pdf, 4.0, 2.0;
gamma_Q, normal_pdf, 0.4, 0.2;
stderr e_R, inv_gamma_pdf, 0.4, inf;
stderr e_g, inv_gamma_pdf, 1.0, inf;
stderr e_z, inv_gamma_pdf, 0.5, inf;
end;
```

Priors plot (I)



Priors plot (II)



Dynare Estimation (VI)

```
varobs YGR INFL INT;

// computing posterior mode
estimation(datafile=as_data,first_obs=50,nobs=250,
           mh_replic=0,mode_check);
```

Dynare Output (I)

```
Initial value of the log posterior
                   (or likelihood): -812.3343
-----------------
f at the beginning of new iteration, 812.3343
Predicted improvement:      463.840722155
lambda =           1; f =       30459.8501185
lambda =     0.33333; f =        1092.5770840
lambda =     0.11111; f =         752.7353413
Norm of dx     0.30458
----
Improvement on iteration 1 =       59.598924800
...
Improvement on iteration 42 =        0.000000003
improvement < crit termination

Final value of minus the log posterior
                       (or likelihood):590.429395
```

Dynare Output (II)

```
RESULTS FROM POSTERIOR ESTIMATION
parameters
    prior mean     mode    s.d. prior pstdev

tau       2.000   2.0854  0.3799 gamm   0.5000
kappa     0.200   0.1697  0.0253 gamm   0.1000
psi_1     1.500   1.4385  0.2320 gamm   0.2500
psi_2     0.500   0.3303  0.1226 gamm   0.2500
rho_R     0.500   0.5613  0.0422 beta   0.2000
rho_g     0.800   0.9529  0.0169 beta   0.1000
rho_z     0.666   0.5736  0.0290 beta   0.1500
r_A       0.500   0.6574  0.2722 gamm   0.5000
pi_A      4.000   3.9657  0.0338 gamm   2.0000
gamma_Q     0.400   0.3744  0.0762 norm   0.2000
```

Dynare Output (III)

```
standard deviation of shocks
    prior mean     mode    s.d. prior pstdev

e_R      0.400   0.1757  0.0093 invg     Inf
e_g      1.000   0.7684  0.0342 invg     Inf
e_z      0.500   0.5492  0.0451 invg     Inf


Log data density [Laplace approximation] is -621.416125.
```

Simulating the posterior distribution

- The posterior distribution doesn't have an analytical representation.
- It is possible to simulate a sample of parameter values, representative of the posterior distribution.
- Metropolis is a MCMC algorithm with acceptance/rejection of proposed parameter va ues randomly drawn.
- The average acceptance must be monitored. A target of about 25% is considered optimal.
- The average acceptance ratio can be tuned with the **mh_jscale** parameter of the **estimation** command.

Dynare command

```
estimation(datafile=as_data,first_obs=50,nobs=250,
           mode_compute=0,mode_file=as_est_mode,
           mh_replic=100000,mh_nblocks=1,mh_jscale=0.7,
   fast_kalman_filter,
   bayesian_irf,forecast=12);
```

Smoothed variables and smoothed shocks

- Only some variables are observed (here YGR INFL INT).
- The Kalman smoother finds best values for other variables and for shocks.
- In absence of measurement errors, the smoothed value of the observed variables is equal to the observation.

Dynare output (I)

```
Estimation::mcmc: Current acceptance ratio per chain:
                                          Chain  1: 21.08%
ESTIMATION RESULTS

Log data density is -621.301701.
```

Dynare output (II)

parameters

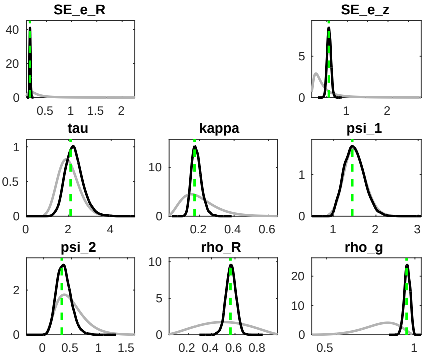
	prior mean	post. mean	90% HPD interval	
tau	2.000	2.2520	1.5927	2.9147
kappa	0.200	0.1815	0.1363	0.2274
psi_1	1.500	1.4870	1.0970	1.8468
psi_2	0.500	0.3673	0.1619	0.5713
rho_R	0.500	0.5651	0.4986	0.6323
rho_g	0.800	0.9559	0.9295	0.9824
rho_z	0.666	0.5773	0.5297	0.6250
r_A	0.500	0.6813	0.2418	1.1119
pi_A	4.000	3.9629	3.9110	4.0173
gamma_Q	0.400	0.3676	0.2473	0.4929

Dynare output (III)

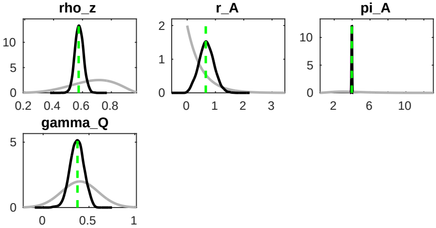
```
standard deviation of shocks
        prior mean  post. mean       90% HPD interval

e_R          0.400      0.1782     0.1630      0.1940
e_g          1.000      0.7777     0.7208      0.8352
e_z          0.500      0.5530     0.4755      0.6275
```

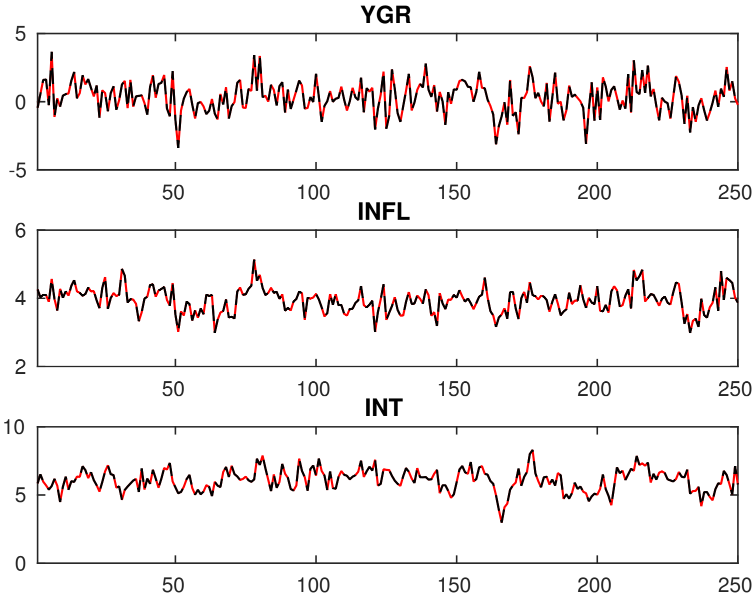
Prior-Posterior-Mode (I)



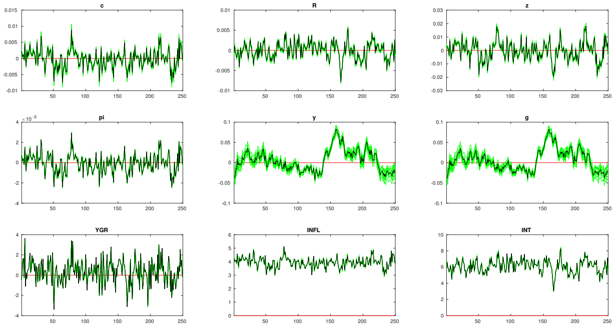
Prior-Posterior-Mode (II)



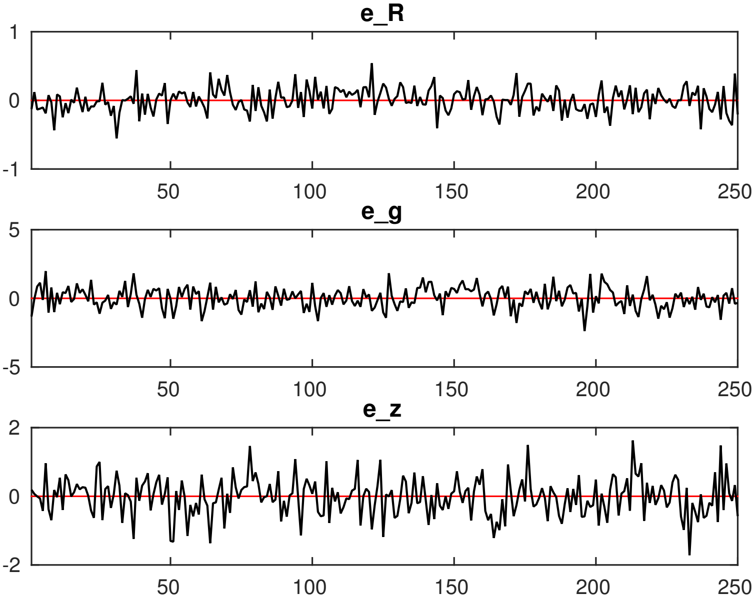
Smoothed observed variables



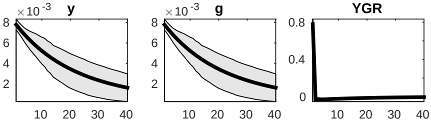
Smoothed variables



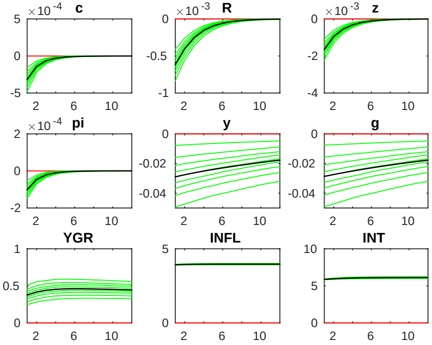
Smoothed shocks



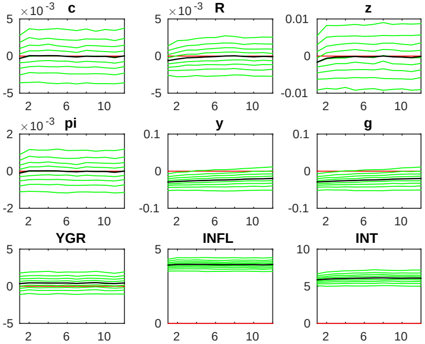
Posterior distribution of IRFs



Posterior distribution of forecast mean



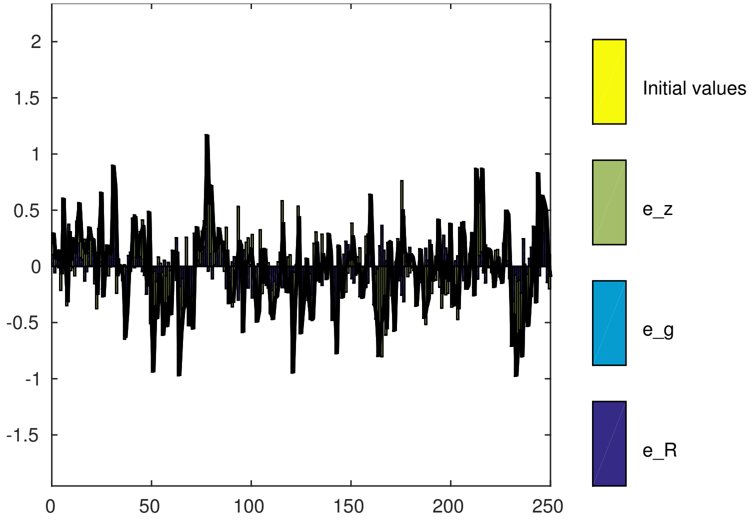
Posterior distribution of forecast point



Historical shocks decomposition

- Because the reduced form of the model is a state space, it is possible to decompose the dynamics of each endogenous variables as the sum of initial conditions and the cumulative effects of the shocks
- In real applications, this decomposition can be compared to a narrative of events.

Historical shock decomposition of inflation



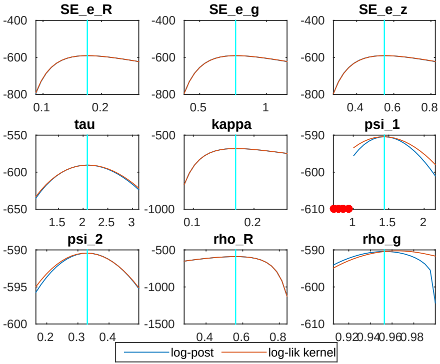
Points to watch for

- Numerical optimization for finding the mode not always works:

```
POSTERIOR KERNEL OPTIMIZATION PROBLEM!
 (minus) the hessian matrix at the "mode" is not positive definite!
=> posterior variance of the estimated parameters are not positive.
You should try to change the initial values of the parameters using
the estimated_params_init block, or use another optimization routine.
Warning: The results below are most likely wrong!
```

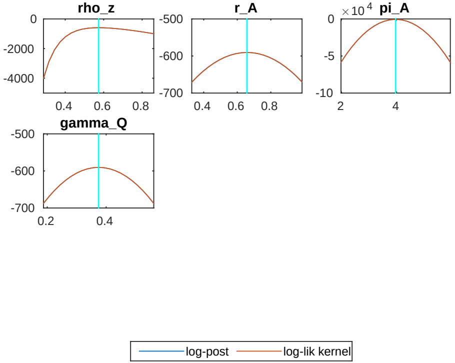
- Check for parameter standard deviation of 0
- If needed use option `mode_check`

Mode check



SE_e_R
SE_e_g
SE_e_z
tau
kappa
psi_1
psi_2
rho_R
rho_g
log-post
log-lik kernel

Mode check


rho_z
0
-2000
-4000
0.4
0.6
0.8
r_A
-500
-600
-700
0.4
0.6
0.8
×10 4
pi_A
0
-5
-10
2
4
gamma_Q
-500
-600
-700
0.2
0.4
log-post
log-lik kernel

Points to watch for

- Numerical optimization for finding the mode not always works:

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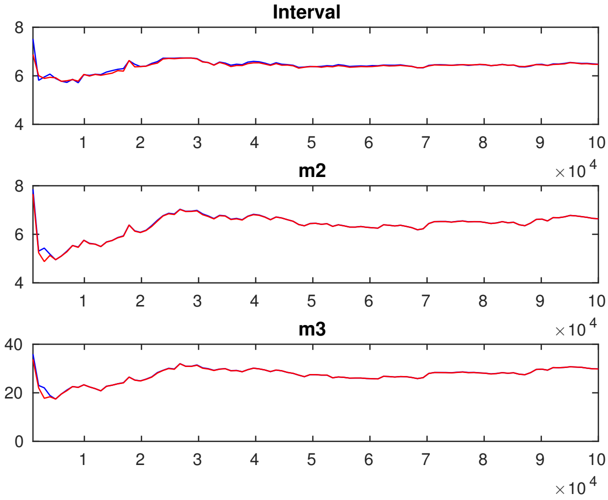
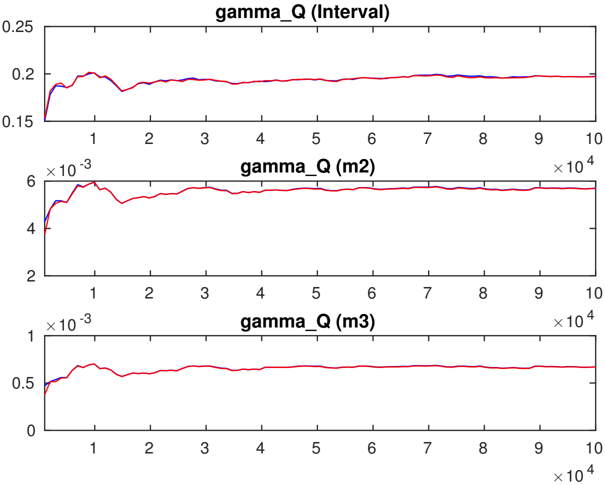
```

- Check for parameter standard deviation of 0
- If needed use option `mode_check`
- Assess convergence of Metropolis sampling: Brooks and Gelman (1998) convergence tests

Multivariate convergence tests



Univariate convergence tests



More advice

- Data must be consistent with the variables of the model (stationarity units)
- Check that a model simulated under the prior produces moments roughly comparable with the empirical moments of the data
- Start with a small model and add mechanisms one by one. This makes it easier to find mistakes. You can use better initial values for the parameters.